School Level

Probability

Probability में हमको बहुत सारी terminologies सीखना है, एक बार के हम यह सारी terminologies सीख गए तो हम questions करेंगे।

(1.) Random Experiment :-

ऐसा experiment जिसमें एक ही event को बार बार perform करने पर different output मिले उसे Random Experiment कहते हैं।

eg: Tossing a coin, Throwing a dice, picking a card.

⇒ Random experiment किसी भी event का उसके पास एक list of possible outcomes होता है, उसी outcomes के अंदर randomness होगा।

e.g:- outcomes for event Tossing a coin — {H, T}
outcomes for event Throwing a dice — {1, 2, 3, 4, 5, 6}

(ii.) Sample Space :

Sample space वह टोकरी है जिसमें वह सारे outcomes रखे होते हो जो उस event से आ सकते है।

e.g: Sample space for tossing a coin
{H, T}

Throwing a dice → {1, 2, 3, 4, 5, 6}
Picking a card → 52 outcomes

Q write sample space for following events?

(i) I'm throwing a dice till a six comes out/ shows up.

इस event का sample space होगा

तो अगर तुम्हारा answer 6 or 5 or 1 है तो यह सारे answers गलत है।

तो कोशिश करते हैं इसका सही sample space निकालने का कोशिश करेंगे

पहली बार dice throw किया तो '6' आया
या first throw मे '1' आया और second throw मे '6' आया
या first throw मे '2' आया और second throw मे '6' आया
या first throw मे '3' आया और second throw मे '6' आया
वैसे हो (4, 6), (5, 6) आएगा।

लेकिन ये किसने बोला की दूसरी throw मे 6 आ जाएगा, अगर second throw मे 6 नही आता तो हमको

third बार dice फेंकना पड़ेगा
जैसे (1,1,6) आ सकते हैं, (1,2,6), (1,3,6), (1,4,6), (1,5,6) आ सकते हैं और अभी और भी तो आएगा (2,1,6) (2,2,6)
अब आपको लगभग question समझ मे आया होगा, मैं आपसे फिर पूछता हूँ की इसके sample space मे कितना elements होगा → Infinite elements होगा।
तो third मे outcome मे 6 आना है तो मैं बहुत सारा elements लिख सकता हूँ इसके बाद मुझे क्या लिखना पड़ेगा fourth outcome मे 6 आए जैसे
(1,1,1,6), (1,1,2,6) (1,1,3,6) - - - - -
(1,2,1,6), (1,2,2,6) (1,2,3,6) - - - - -
ऐसा बहुत सारा outcomes आएगा।

(ii) event of throwing a dice if a '6' shows up then i will throw a coin otherwise i will throw the dice again.

Sample Space

{
(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,H) (6,T)
}

This sample space will have '32' elements.

Sample space मे दो तरह का sample space होते है :-

(i) Fundamental sample space
(ii) Derived sample space

For example के लिए लिया :-

Tossing two dice.

इसके लिए sample space होगा

S.S = { 11, 12, 13, 14, 15, 16 ...
21, 22, - - - - -
:
:
61, 62, 63, - - - - - - 66 }

↳ तो इस sample space मे मेरे पास 36 outcomes है

⇒ अगर मान लो same experiment कर रहे है लेकिन हम sum of the outcomes of the two dice को observe कर रहे हो you are not observing the individual outcomes.

for example मैं दो dice को toss कर रहा हूँ यहाँ जो outcome होगा वो आपको दिखेगा नहीं obviously you are sitting far beyond where this experiment is happening.
So in order की आपको outcome दिखे मैं एक screen पर दो dice के outcome का sum display करदेता हूँ।

So, what are the possible outcomes तो sum जो है अगर outcome (1,1) है तो 2 आ सकते है, वैसे ही (1,2) है तो 3 आ सकते ; (1,3) है तो 4 आ सकते ; (1,4) है तो 5 आ सकते कहाँ तक जा सकते है 12 तक जा सकते है तो अपने हिसाब से what are the outcomes 2, 3, 4, upto ... 12 तो हमारे लिए sample space क्या होगा यही होगा ना।

If only sum is being observed

$= \{2, 3, 4, 5, \ldots\ldots, 12\}$

अब question है कि पहली बार जब हम सिर्फ individual outcomes को consider कर रहे थे तो जो sample space आया और अब जब sum of the dice को as a outcome consider करके उसका sample space निकाला तो उसका जो sample space निकला तो दोनो मे क्या difference है।

अब देखो

$S_1 = \{ (1,1), (1,2), (1,3), (1,4) (1,5), (1,6)$
(2,1)
⋮ $\}$

↳ यह fundamental sample space है

और

$S_2 = \{ 2, 3, 4, 5, \ldots\ldots 12 \}$
↳ Derived sample space

अब मान लो कि मे S_1 मे हो हर एक element के individual possibility की बात करे तो वह same होगा। i.e. all elements have equal likely to come.
कैसे समझो, आपने bet किया कि (1,1) outcome आएगा और बगल वाला bet किया की (6,6) outcome आएगा; दोनो जीतने का probability same है या अलग - अलग है, same है ना क्योंकी जितना (1,1) आने का chance है; (6,6) आने का ही उतना ही chance है।

लेकिन क्या S_2 sample space मे possibility of every element is same होगा क्या?

मान लिजिए one of you have placed your bet that sum 2 होगा, दूसरा आदमी bet किया की sum 7 होगा तो क्या दोनों का probability same है क्या

sum is 2 (1,1) → 1 outcome
sum is 7 (1,6) (2,5) (3,4) → 6 outcomes
(4,3) (5,2) (6,1)

Clear; है की probability की sum 7 आए उसका ज्यादा होगा जब sum 2 आए उससे

इसका मतलब यह हो गया कि S_2 sample space के elements के आने का possibility is not equally likely.

In a nutshell :—

☆ Fundamental sample space :— मे elements के आने कि possibility (each) same / equally likely होती है।

☆ Derived sample space :— मे elements के आने कि possibility same नहीं / not equally likely होती है।

Event

→ event का physical meaning यह होता है कि "a happening on which you can place your bet."

→ usually probability deals with betting, most common use of probability is in betting.

→ मान लिजिए "event" is some happening on which bet can be placed.

e.x: एक event है throwing a dice.

इसके लिए sample space है
{1, 2, 3, 4, 5, 6}

अब time is for betting.

So what are different types of betting?

जैसे
6 पर bet किया
5 पर bet किया
4 पर bet किया
3 पर bet किया
2 पर bet किया
1 पर bet किया
⋮

So what are the different ways you can bet each of them is called an event

so $E_1 = \{1\}$ → जब 1 पर betting कर रहे है

$E_2 = \{2\}$ → जब 2 पर betting कर रहे है

तो अब तुमको लग रहा होगा की इस event मे सिर्फ 1-6 यानी 6 तरीके से ही betting हो सकता है पर ऐसा जरूरी नही है क्योंकी even/odd no. पर भी तो कर सकते है वैसे ही बहुत तरीके से betting कर सकता हूं।

$E_3 = \{3\}$

$E_4 = \{2, 4, 6\}$

$E_5 = \{1, 3, 5\}$

$E_6 = \{2, 4, 5, 6\}$

So there can be different ways of betting; but sum of these bettings can be expressed in english also

देखिए जैसे E_4 की बात करे तो mathematically मे बोल सकता हूं की i am betting on $\{2, 4, 6\}$ पर english मे भी मैं बोल सकता।

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है की i am betting on a even number
लेकिन E_6 को हम english मे नही बोल पाएंगे
तो कभी कभी english मे बोलना मुश्किल भी पड़ सकता है
इसलिए सबको english मे convert करना मुश्किल हो सकता है
that's why think mathematically ये बहुत महत्वपूर्ण बात है

तो मान लो अगर मैं बोलूं तुमसे की i have placed my bet on $E_4 = \{2, 4, 6\}$.
तो मैं तुमको दो sentence बोलता हूं

(a) you have placed your bet on 2 (or) 4 (or) 6.

(b) you have placed your bet on 2 (and) 4 (and) 6.

'or' वाला सही है।
क्योंकि अगर 'and' वाला सही है तो मैं जब एक dice throw किया तो 2 भी आना चाहिए 4 भी आना चाहिए & 6 भी आना चाहिए

आ सकता है जैसे एक dice throw करने पर तीनों 2, 4 & 6 आए नहीं आएगा ना!
या तो 2 आएगा या 4 आएगा या 6 आएगा।

इसलिए अगर मैं पूछूँ तुमसे की what is the meaning of event happening (let's say for E4) तो उसका मतलब है की या तो 2 आए या 4 आए या 6 आए तो आप जीत जाएँगे। i.e the event has happend.

event not happening का मतलब होगा की ना तो 2 आया ना 4 ना 6 आया।

तो event का मतलब क्या है?
Some happening on which bet can be placed.

लेकिन यह तो बहुत अजीब सा definition लग रहा है की event define करने के लिए betting क्यों करना है मुझे।

So what is the mathematical definition of event is:
⇒ a subset of Sample space

तो अब हमे जानना है कि एक event में कितने elements हो सकते है।

i.e. No. of events $= 2^n$

where $n = n(S) \rightarrow$ no. of elements in sample space

तो अभी हमने सीखा की सारे events जो है, वह sample space के subset होते है और हमने set theory chapter में पढ़ा था की एक set के 2^n subsets होंगे, where n is no. of elements in the set.

e.g; How many events are there for tossing a coin

a coin has two outcomes {H, T}

therefore,

No. of events $= (2)^2$
$= 4$ events.

एक बार देखिए tossing a coin के लिए sample space जो होगा {H, T}.

अब what are the possible events :- तो हमको formulae से पता लगा कि 2^2 यानी 4 event होंगे।

Events → ϕ, {H}, {T}, {H, T}

तो पहला event होगा ϕ ; मैं ऐसा betting करता हूँ कुछ भी नहीं आएगा हो सकता है ऐसा कौन मना करता है मुझे हर बार हार जाऊँगा जैसे मैं betting house में गया और मैंने betting किया कि अगर कुछ नहीं आया तो मैं 10 हजार दूँगा।

दूसरा event होगा head आएँ।

तीसरा event होगा tail आएँ।

चौथा event होगा head और tail दोनों आएँ यानी कुछ भी आए तो मैं जीतूँगा यानी पूरा sample space में betting कर सकता हूँ।

तो थोड़ा सा अजीब लग रहा है इसलिए नामकरण कर देंगे & events को

ϕ ~~& {H, T}~~ → को बोलेंगे "Impossible event"

क्योंकी यह कभी नही होगा

{H,T} → को बोलेंगे "sure event" क्योंकी यह तो होना ही है अगर एक coin को toss करेंगे तो या तो head आएगा या tail आएगा।

और ϕ & {H,T} को combinedy बोलेंगे trivial event बोलेंगे very obvious क्योंकी हमको set theory से याद होगा की subset के topic मे हमने पढ़ा था की empty set or set itself हर set का subset होता अगर यहाँ हम ध्यान दें तो ϕ यानी की impossible event और {H,T} यानी की sure event trivial event है।

और बाकी event sure & impossible event को छोड़ कर उनको non-trivial events बोलेंगे।

Non-Trivial events मे अब बहुत प्रकार के event होंगे जिसे हम types of events मे पढ़ेंगे।

Types of Events

① Elementary or Simple Event :-

ऐसा event जिसमे सिर्फ एक outcome आए।

(या) single element पर betting किया।

e.g: Throwing a dice में 1 पर betting किया (या)
2 पर ——— (या)
③ पर ———
so on... ④ or 5 or 6.

* Elementary event = $n(E) = 1$.

② Compound or Mixed Event :-

ऐसा event जिसमे एक से ज्यादा outcome आए।

(या) एक से ज्यादा element पर जब betting हो।

e.g: Throwing a dice में 2 (या) 3 पर betting किया

(P.T.O.)

③ Complementary Event :—

Represented by $\bar{E}$ (or) E^c (or) $\bar{E}$

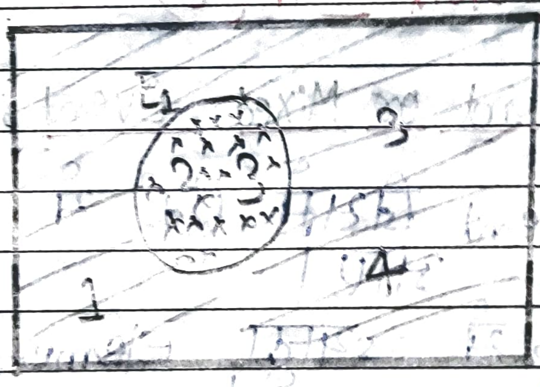
या इसको बोलेंगे not E

↳ Sample space − E

e.g.: Sample space = {1, 2, 3, 4}

$E_1 = \{2, 3\}$

$\bar{E_1} = \{1, 3, 4\}$



 → Shows $\bar{E_1}$

 → Shows E_1

(P.T.O.)

4. Mutually Exclusive events :—

देखिए अभी ③ तक event यानी single event का बात हो रहा था अब events हो गया यानी एक से ज्यादा event आ जाएँगे consideration में।

Two events are said to be exclusive if they don't have any intersection.

जब दो events के बीच कोई भी element common ना हो तो उसे Mutually exclusive events बोलेंगे।

e.g.: $E_1 = \{1, 2, 3\}$ $E_2 = \{5, 6\}$

$E_1 \cap E_2 = \phi$ so,

E_1 & E_2 are mutually exclusive events.

* तीन events भी mutually exclusive हो सकते हैं; कभी-कभी questions में आएगा three events are pairwise mutually exclusive, इसका मतलब क्या है

$E_1 \cap E_2 = \phi$

$E_2 \cap E_3 = \phi$

$E_1 \cap E_3 = \phi$

(5) Exhaustive Events :-

⇒ "exhaust" का मतलब होता है खत्म हो जाना।

If you have some no. event let's say $E_1, E_2, E_3, \ldots, E_n$ such that,

$$E_1 \cup E_2 \cup E_3 \ldots \cup E_n = S$$

where,
S ↦ sample space

ये सारा events combine कर के अगर sample space आ जाए then such events are called "Exhaustive Events"; they are spread through complete sample space; इसका मतलब कुछ भी outcome आया some of you will win.

ex: E_1 एक event है {1,2}
E_2 दूसरा event है {2,3}
$S = \{1, 2, 3, 4, 5, 6\}$ E_3 तीसरा event है {3, 4, 5}
E_4 चौथा event है {1, 6}

अब देखो ये चारो event मिल कर पूरा sample space cover करता है।

Algebra of Events

① E_1 and E_2 :-

जब भी probability मे हम यह बोलेंगे कि E_1 and E_2 तो उसको ऐसे बोलेंगे :-
"$E_1 \cap E_2$"

② Exactly one of E_1 or E_2 :-

जब भी probability मे हम यह बोलेंगे कि exactly one of E_1 or E_2 तो उसका मतलब होता है कि
E_1 winning and E_2 not winning
(or) E_1 not winning and E_2 winning
तो उसको ऐसे बोलेंगे :-

$$``(E_1 \cap \overline{E_2}) \cup (\overline{E_1} \cap E_2)"$$

③ E_1 but not E_2 :-

जब भी probability मे हम यह बोलेंगे कि E_1 but not E_2 तो उसका मतलब होता है कि

$$``(E_1 \cap \overline{E_2})"$$

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(4) Exactly one of E_1, E_2 or E_3 :—

probability में exactly one of E_1, E_2 or E_3 का मतलब होगा कि

E_1 wins and E_2 losses and E_3 losses (or)
E_1 losses and E_2 wins and E_3 loses (or)
E_1 loses and E_2 loses and E_3 wins.

इसका मतलब होगा :→

"$(\overline{E_1} \cap E_2 \cap \overline{E_3}) \cup (E_1 \cap \overline{E_2} \cap \overline{E_3}) \cup (\overline{E_1} \cap \overline{E_2} \cap E_3)$"

(5) Neither E_1 nor E_2 :—

probability में neither E_1 nor E_2 का मतलब होगा कि

"$(\overline{E_1} \cap \overline{E_2})$"

Formulaes :—

☆ $P(E_1 \cup E_2) = P(E_1) + P(E_2)$; if $E_1 \cap E_2 = \phi$

देखिए ऊपर वाला formulae यह बोल रहा है कि $P(E_1 \cup E_2) = P(E_1) + P(E_2)$ तब होगा जब E_1 and E_2 are mutually exclusive.

* $P(\overline{E}) = 1 - P(E)$ {Complement Law}

जैसे मान लिजिए एक event का happening probability $\frac{1}{3}$ है, तो not happening probability कितना होगा $1 - \frac{1}{3} = \frac{2}{3}$. तो कोई event का

अगर probability पता है तो उसका not happening का probability हम आसानी से निकाल सकते हैं Complement Law के उपयोग से।

* $P(\overline{E_1} \cap \overline{E_2}) = P(\overline{E_1 \cup E_2}) = 1 - P(E_1 \cup E_2)$

$P(\overline{E_1} \cup \overline{E_2}) = P(\overline{E_1 \cap E_2}) = 1 - P(E_1 \cap E_2)$

{De-Morgan's Law}

→ इसका मतलब है probability of neither E_1 not happening nor E_2 not happening.

→ इसका मतलब है probability of either E_1 not happening or E_2 not happening.

✯ Odds in favour of E :-

Odds in favour of E का मतलब होता है :

$$\text{odds in favour of } E = \frac{P(E)}{P(\bar{E})} = \frac{P(E)}{1 - P(E)}$$

i.e., probability of event happening divided by probability of event not happening.

✯ Odds ~~in~~ against E :-

Odds ~~in f~~ against of E का मतलब होता है :

$$\text{odds against of } E = \frac{P(\bar{E})}{P(E)} = \frac{1 - P(E)}{P(E)}$$

i.e., probability of event not happening divided by probability of event happening.

Probability of an Event

मान लो कि एक bag में 90 red and 10 white balls है जो की shape and size में similar है।

अगर सब balls को मिला दिए और उसके बाद एक ball को draw किए at random तो वह या तो red होगा या white.

साफ बात है कि जो ball draw होगा उसकी red आने की संभावना ज्यादा है white से।

तो हम उसे ऐसे बोलेंगे की the event of drawing a red ball is more probable than the event of drawing a white ball.

जितने भी event जो random experiment से associated है हम कोशिश करेंगे कि एक numerical value उसके साथ जोड़ें जिसे "probability" कहते हैं।

in such a manner that for any two events, the event which is more likely has a higher probability.

Probability of an Event

In a random experiment, Let S be the sample space and let $E \subseteq S$. Then E is an event.

The probability of occurence of E is defined as

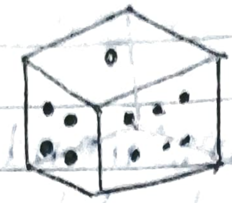
$$P(E) = \frac{\text{no. of outcomes favourable to occurence of } E}{\text{no. of all possible outcomes}}$$

$$= \frac{\text{no. of distinct elements in } E}{\text{no. of distinct elements in } S}$$

$$= \frac{n(E)}{n(S)}$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)}$$

☆ Dice



- A die is a solid cube
- It has six faces that are marked with 1, 2, 3, 4, 5 and 6 dots.
- In throwing a die, the outcome is the number of dots on the uppermost face.

☆ Playing Cards

	Clubs	Diamonds	Hearts	Spades
Ace	1	1	1	1
2	1	1	1	1
3	1	1	1	1
4	1	1	1	1
5	1	1	1	1
6	1	1	1	1
7	1	1	1	1
8	1	1	1	1
9	1	1	1	1
10	1	1	1	1
Jack	1	1	1	1
Queen	1	1	1	1
King	1	1	1	1
	13 cards	13 cards	13 cards	13 cards

→ A standards 52- card deck comprises

suits
- clubs — 13 cards
- diamonds — 13 cards
- hearts — 13 cards
- spades — 13 cards

→ Each suit includes an Ace, a King, Queen and Jack.

Results on Probability :-

Let S be the sample space & let E be an event. Then,

(i.) $P(E) \geq 0$

किसी भी event का probability अगर वह impossible event नहीं है तो zero से ज्यादा होगा कभी भी -ve नहीं होगा।

(ii.) $P(\phi) = 0$

impossible event का probability zero होता है।

(iii.) $P(S) = 1$

sure event का probability 1 होता है।

Addition Theorem

☆ For any two events E_1 and E_2,

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

i.e.,

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2)$$

☆ For any three events E_1, E_2 & E_3.

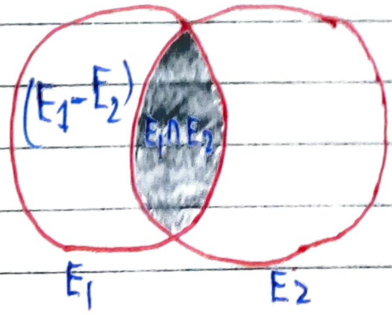
$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_1 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

NOTE:- If E_1 and E_2 are two mutually exclusive events then

$$P(E_1) + P(E_2) = 1.$$

★ For any two compatible events E_1 and E_2,

$$P(E_1 - E_2) = P(E_1) - P(E_1 \cap E_2)$$



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★ If E_1 and E_2 be two events such that $E_1 \subseteq E_2$ then $P(E_1) \leq P(E_2)$.

Q. A die is thrown. Find the probability of :-

(i) getting a 5
(ii) getting a 2 or a 3
(iii) getting an odd number
(iv) getting a prime number
(v) getting a multiple of 3
(vi) getting a number between 3 and 6.

Ans

$S = \{1, 2, 3, 4, 5, 6\}$
Total no. of outcomes = 6

(i)

$$P(\text{getting a } 5) = \frac{\text{no. of outcomes favourable to getting a } 5}{\text{total no. of outcomes}}$$

$$= \frac{1}{6}$$

(ii)

P(getting a 2 or 3)

$$= \frac{\text{no. of outcomes favourable to getting a 2 or 3}}{\text{total no. of outcomes}}$$

$$= \frac{2}{6} = \frac{1}{3}$$

(iii.) P(getting an odd number)

$$= \frac{\text{no. of outcomes favourable to getting an odd number}}{\text{total no. of outcomes.}}$$

$$= \frac{\cancel{2}}{\cancel{6}_3} = \frac{1}{3}$$

(iv.) P(getting a prime number)

$$= \frac{\text{no. of outcomes favourable to getting a prime number}}{\text{total no. of outcomes}}$$

$$= \frac{\cancel{3}}{\cancel{6}} = \frac{1}{2}$$

(v.) P(getting a multiple of 3)

$$= \frac{\text{no. of outcomes favourable to getting a multiple of 3}}{\text{total no. of outcomes}}$$

$$= \frac{\cancel{2}}{\cancel{6}} = \frac{1}{3}.$$

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(vi.) getting a number between 3 and 6

अब देखो b/w 3 & 6 का मतलब होता है कि 3 और 6 के बीच 3 और 6 को छोड़कर i.e. 5 and 4.

P(getting a no. between 3 and 6)

$$= \frac{\text{no. of outcomes favourable to getting a no. b/w 3 \& 6}}{\text{total no. of outcomes}}$$

$$= \frac{2}{6} = \frac{1}{3}.$$

Q In a single throw of two dice, find the probability of :-

(i.) getting a sum less than 6
(ii.) getting a doublet of odd numbers.
(iii.) getting the sum as a prime number.

(P.T.O.)

Ans: question में दीया है कि हमने दो dice को एक बार फेंका

Total no. of outcomes = 6×6 = 36 outcomes

(i) getting a sum less than 6

(1,4), (1,3), (1,2), (1,1)
(2,1), (2,2), (2,3), (3,2),
(3,2) (4,2)

10 outcomes favourable

P(getting a sum less than 6)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}} = \frac{10}{36} = \frac{5}{18}$

(ii) getting a doublet of odd numbers

"doublet" का मतलब होता है pair of two similar elements.

(1,1), (3,3), (5,5)

P(getting a doublet of odd numbers)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{3}{36} = \frac{1}{12}$

(iii) getting the sum as a prime number

(1,1), (1,2), (1,4), (1,7)
(2,1), (2,3), (2,5)
(3,2), (3,4).
(4,1), (4,3)
(5,2), (6,1) (6,5)
(5,6)

} 15 favourable outcomes

P(getting the sum as a prime no.)

$$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{\cancel{15}\ 5}{\cancel{36}\ 12} = \frac{5}{12}$$

Q In a single throw of two dice, find:-

(i) P(an odd no. on the first die and a 6 on the second)
(ii) P(a number greater than 3 on each die)
(iii) P(a total of 10)
(iv) P(a total greater than 8)
(v) P(a total of 9 ~~or~~ ~~11~~)

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Ans: Sample space has = 6 × 6
= 36 outcomes

(i) Probability of an odd no. on the first die and a 6 on the second die

(1, 6), (3, 6), (5, 6) } 3 favourable outcomes.

P(getting an odd no. on the first die and a 6 on the second die)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}} = \frac{3}{36} = \frac{1}{12}$

(ii) Probability of getting a number greater than 3 on each die

(4, 4), (4, 5), (4, 6)
(5, 4), (5, 5), (5, 6)
(6, 4), (6, 5), (6, 6)
} 9 favourable outcomes

P(getting a no. greater than 3 on each die)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}} = \frac{9}{36} = \frac{1}{4}$

(iii.) Probability of getting a total of 10

(4,6), (6,4), (5,5) } 3 favourable outcomes

P(getting a total of 10)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{3}{36} = \frac{1}{12}$

(iv.) Probability of getting a total greater than 8

(3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6) } 10 favourable outcomes

P(getting a total greater than 8)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{10}{36} \frac{5}{18} = \frac{5}{18}$

(v.) Probability of getting a total of 9 or 11

$(3,6), (4,5), (5,4), (6,3)$ } 4 favourable outcomes

P(getting a total of 9 or 11)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}} = \frac{4}{36} = \frac{1}{9}$.

Q. A bag contains 4 white and 5 black balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is white.

Ans: (4 white balls) (5 black balls)

No. of ways of selecting one white ball out of 4 available white balls

$= {}^{4}C_1 = 4$

P(getting a ball drawn is white)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{4}{9}$

Q. In a lottery, there are 10 prizes and 25 blanks. Find the probability of getting a prize.

Ans:

Total no. of outcomes

$= 10 + 25$

$n(S) = 35$

P (getting a prize)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{\cancel{10}\,2}{\cancel{35}\,7} = \frac{2}{7}$

Q. If there are 2 Children in a family, find the probability that there is at least one boy in the family?

Ans:

Total outcomes

$= \{(B,B), (B,G), (G,B), (G,G)\}$

(first three marked: favourable outcomes)

(P.T.O.)

So,

P(at least one boy in the family)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{3}{4}$

Q Three unbiased coins are tossed once. Find the probability of getting

(i.) exactly two tails.
(ii.) exactly one tail
(iii.) at most 2 tail.
(iv.) at least 2 tail
(v.) at most 2 tail or at least 2 head.

Ans:

Total outcomes

= {(H,H,H) (H,H,T) (H,T,H)
(H,T,T) (T,H,H) (T,H,T)
(T,T,H) (T,T,T)}

(i) exactly two tails :-

favourable outcomes
= {(H,T,T), (T,H,T) (T,T,H)}

So,

P(exactly two tail(s))

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{3}{8}$

(ii) exactly one tail :-

favourable outcomes

= {(H,H,T), (T,H,H)}

So, P(exactly ~~two~~ one tails)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{2}{8} = \frac{1}{4}$

(iii) at most two tail :-

favourable outcomes :

= {(H,H,H), (H,H,T), (H,T,H), (H,T,T), (T,H,H), (T,H,T), (T,T,H)}

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so, P(at most two tails)

$$= \frac{\text{favourable outcomes}}{\text{total outcomes}} = \frac{7}{8}$$

(iv.) at least two tail :—

favourable outcomes

$= \{ (H, T, T), (T, H, T), (T, T, H)$
$(T, T, T) \}$

so, P(at least two tails)

$$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{4}{8} = \frac{1}{2}.$$

(V.) at most 2 tail or at least 2 head :—

favourable outcomes

$= \{ (H, H, H), (H, H, T), (H, T, H)$
$(T, H, H), \}$

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So,

P(at most 2 tail or atleast 2 head)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{4}{8} = \frac{1}{2}$

Q. In a single throw of dice, determine the probability of not getting the same number on the two dice.

Ans:

total outcomes $= 6 \times 6$
$= 36$ outcomes

$= \{ (1,1) \quad (2,2) \quad (3,3) \quad (4,4) \quad (5,5) \quad (6,6) \}$

So,

P(getting same no. on two dice)

$P(E) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{6}{36} = \frac{1}{6}$

Now,

P(not getting same no. on the two dice)

$P(\bar{E}) = 1 - P(E)$

$= 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$.

Q. If a letter is chosen at random from the English alphabet, find the probability that the letter chosen is :–
(i.) a vowel (ii.) a consonant

Ans: No. of letters in english alphabet = 26
$n(S) = 26$

(i.) a vowel

We know there are five vowels namely a, e, i, o, u, total 5 favourable outcomes

$$P(\text{getting a vowel}) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{5}{26}$$

(P.T.O.)

(ii) a consonant

we know there are 21 consonants.

P(getting a consonant)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{21}{26}$

Q. A card is drawn at random from a well-shuffled pack of 52 cards. What is the probability that the card bears a number greater than 3 and less than 10?

Ans: total outcomes = 52 cards

Numbers would be
↳ {4, 5, 6, 7, 8, 9} (6)

And we know there are four suits

= 6 x 4
= 24 cards

∴ P(card bears a number greater than 3 and less than 10)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{\cancel{24}\ \cancel{12}\ 6}{\cancel{52}\ \cancel{26}\ 13} = \frac{6}{13}$

Q. Tickets numbered from 1 to 12 are mixed up together and then a ticket is withdrawn at random. Find the probability that the ticket has a number which is a multiple of 2 or 3.

Ans: $n(S) = 12$.

favourable outcomes

$= \{2, 4, 6, 8, 10, 12, 3, 9\}$. → total 8 outcomes

P(ticket has a number which is a multiple of 2 or 3)

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{\cancel{8}\ \cancel{4}\ 2}{\cancel{12}\ 6} = \frac{2}{3}$.

Q. What is the probability that an ordinary year has 53 Tuesdays?

Ans:

No. of days in a year = 365

52 weeks + 1 days

अब एक week में 7 दिन होते है इसलिए total outcomes / sample space होगा :—

$m(S) = 7$

तो हमे क्या चाहिए tuesday चाहिए है इसलिए no. of favourable outcomes होगा :—

$m(E) = 1$.

Hence

P(of 53 tuesdays) → मतलब उस 7 दिन मे 1 दिन tuesday का हो

$= \frac{1}{7}$.

Q What is the probability that a leap year has 53 sundays?

Ans: No. of days in leap year = 366

52 week + 2 days

the remaing 2 days except 52 weeks can be like :-

{ [Mon, Tues], [Tue., Wed.], [Wed., Thru.], [Thru., Fri.], [Fri., Sat.], [Sat., Sun.], [Sun., Mon.] }

तो इसका मतलब हुआ कि सात कुल favourable outcomes है।

तो हमें क्या चाहिए sunday चाहिए है इसलिए no. of favourable outcomes होगा :-

$n(E) = 1$.

Hence,

P(of 53 sundays)

$= \frac{2}{7}$.

Q. What is the probability that in a group of two people, both will have the same birthday, assuming that there are 365 days in a year and no one has his/her birthday on 29 February?

Ans: A & B are two peoples.

$n(S) = 365 \times 365$ → total outc.

$n(E) = 365$ → favourable outc.

Probability of getting both have the same birthday

$$P(E) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{365}{365 \times 365}$$

$$= \frac{1}{365}$$

Q. If $\frac{7}{10}$ is the probability of occurence of an event, what is the probability that it does not occur?

(P.T.O.)

Ans: Let the event to occur be 'E'

Probability of occurrence of an event

$P(E) = \frac{7}{10}$

Now,

probability that it does not occur

$P(\bar{E}) = 1 - \frac{7}{10} = \frac{3}{10}$

i.e. the probability that it does not occur

$= \frac{3}{10}$

Q. The odds in favour of the occurrence of an event are 8:13. Find the probability that the event will occur?

Ans:

$\therefore$ Odds in favour $= \frac{\text{no. of possibilities in favour}}{\text{no. of possibilities against}}$

$= \frac{8}{13}$

So total no. of possibilities $= 8 + 13$

$= 21.$

$$P(E) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{8}{21}$$

Q. If the odds against the occurence of an event be 4 : 7, find the probability of the occurence of the event.

Ans: let, the probability of event is P(E)

∴ odd against favour occurence of event

$$\Rightarrow \frac{1 - P(E)}{P(E)}$$

$$= \frac{4}{7}$$

$$\Rightarrow 7 - 7P(E) = 4P(E)$$

$$\Rightarrow 7 = 11 \cdot P(E)$$

$$\Rightarrow P(E) = \frac{7}{11}$$

Hence, The probability of occurence of the event $= \frac{7}{11}$.

Q. If $\frac{5}{14}$ is the probability of occurence of an event. find :—

(i) the odds in favour of its occurence.
(ii) the odds against its occurence.

Ans:

Let the P(occurence of an event) be P(E)

$$P(E) = \frac{5}{14}$$

(i) the odds in favour of its occurence.

$$\text{odds in favour} = \frac{P(E)}{1 - P(E)}$$

$$= \frac{5/14}{1 - \frac{5}{14}}$$

$$= \frac{5/14}{\frac{9}{14}} = \frac{5}{9}$$

(P.T.O.)

(ii) the odds against its occurence

$$= \frac{1 - P(E)}{P(E)}$$

$$= \frac{1 - \frac{5}{14}}{5/14} = \frac{\frac{14-5}{14}}{\frac{5}{14}} = \frac{9}{5}$$

Q. Two dice are thrown. find the probability of

(i) the odds in favour of getting the sum 6.

(ii) the odds against getting the sum 7.

Ans:

क्योंकी, दो dice फेंका है इसलिए

total possibilities = 6x6

n(S) = 36.

(i) the odds in favour of getting sum 6

~~favourable~~

favourable outcomes for getting sum as 6

= {(1,5), (2,4), (3,3), (4,2), (5,1)}

P(getting a sum of 6) = $\frac{5}{36}$.

Odds in favour = $\frac{P(\text{getting sum of } 6)}{1 - P(\text{getting sum of } 6)}$

$$= \frac{\frac{5}{36}}{1 - \frac{5}{36}} = \frac{5}{36-5} = \frac{5}{31}.$$

(ii.) <u>the odds against getting the sum 7</u>

favourable outcomes for getting sum as 7

= {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2) (6, 1)}

P(getting a sum of 7) $= \frac{6}{36} = \frac{1}{6}$.

Odds against getting the sum as 7:

$$= \frac{1 - P(\text{getting a sum of } 7)}{P(\text{getting a sum of } 7)}$$

$$= \frac{1 - \frac{1}{6}}{1/6} = \frac{\frac{6-1}{6}}{1/6} = 5.$$

Q. A combination lock on a suitcase has 3 wheels, each labelled with nine digits from 1 to 9. If an opening combination is a particular sequence of three digits with no repeats, what is the probability of a person guessing the right combination?

Ans:

___ ___ ___

(9) (8) (7)

Total no. of lock possible

$= 9 \times 8 \times 7$

$= 72 \times 7$

$= 504$ combinations

Probability of a person guessing the right combination

$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$

$= \frac{1}{504}$ { ∵ there should be only 1 combi. that is the correct lock key. }

Q. If E_1 and E_2 are two events associated with a random experiment such that $P(E_2) = 0.35$, $P(E_1 \text{ or } E_2) = 0.85$ and $P(E_1 \text{ and } E_2) = 0.15$, find $P(E_1)$.

Ans: मान लिया कि $P(E_1) = x$ तब,

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2)$$

$$\Rightarrow 0.85 = P(E_1) + 0.35 - 0.15$$

$$\Rightarrow 0.85 = x + 0.35 - 0.15$$

$$\Rightarrow x = 0.85 - 0.20$$

$$\Rightarrow x = 0.65$$

$$\therefore P(E_1) = 0.65$$

Q. Two dice are tossed together. Find the probability of getting a doublet or a total of 6.

Ans: जब दो dice को throw किए साथ में, तो $(6 \times 6) = 36$ outcomes मिलेंगे

Let S be the sample space.

$$n(S) = 36$$

मान लिया कि

E_1 = event of getting a doublet

E_2 = event of getting a total of 6.

तब

$E_1 = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

and $E_2 = \{(2,4), (3,3), (4,2)\}$

$\therefore (E_1 \cap E_2) = \{(3,3)\}$

इसलिए,

$m(E_1) = 6$; $m(E_2) = 3$

$m(E_1 \cap E_2) = 1$.

$\therefore P(E_1) = \frac{m(E_1)}{m(S)} = \frac{6}{36} = \frac{1}{6}$

$P(E_2) = \frac{m(E_2)}{m(S)} = \frac{3}{36} = \frac{1}{12}$

and

$P(E_1 \cap E_2) = \frac{m(E_1 \cap E_2)}{m(S)} = \frac{1}{36}$

$\therefore$ P(getting a doublet or a total of 6)

$= P(E_1 \text{ or } E_2)$

$= P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$$= \left(\frac{1}{6} + \frac{1}{12} - \frac{1}{36}\right) = \frac{8}{36} = \frac{2}{9}$$

इसलिए required probability होगा $= \frac{2}{9}$

Q. In a single throw of two dice, find the probability that neither a doublet nor a total of 10 will appear.

Ans: Let S be sample space.

तब $n(S) = 36$.

Let, E_1 = event that a doublet appears
& E_2 = event ~~th~~ of getting a total of 10.

तब,

$E_1 = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

and

$E_2 = \{(4,6), (5,5), (6,4)\}$

$\therefore E_1 \cap E_2 = \{(5,5)\}$

Thus, $n(E_1) = 6$, $n(E_2) = 3$
$n(E_1 \cap E_2) = 1$

$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(E_2) = \frac{m(E_2)}{m(S)} = \frac{3}{36} = \frac{1}{12}$$

और,

$$P(E_1 \cap E_2) = \frac{m(E_1 \cap E_2)}{m(S)} = \frac{1}{36}$$

$\therefore$ P(getting a doublet or a total of 10)

$= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \left(\frac{1}{6} + \frac{1}{12} - \frac{1}{36}\right) = \frac{8}{36} = \frac{2}{9}$

$\therefore$ P(getting neither a doublet nor a total of 10)

$= P(\overline{E_1} \text{ and } \overline{E_2})$

$= P(\overline{E_1} \cap \overline{E_2})$

$= P(\overline{E_1 \cup E_2})$

$= 1 - P(E_1 \cup E_2)$

$$= 1 - \frac{2}{9} = \frac{9-2}{9} = \frac{7}{9}.$$

इसलिए, Required probability is $\frac{7}{9}$.

Q. A natural number is chosen at random from among the first 500. What is the probability that the number so chosen is divisible by 3 or 5?

Ans: माना लिया 'S' be sample space.
तो साफ है कि $n(S) = 500$

माना लिया कि

E_1 = event of getting a no. divisible by 3
E_2 = event of getting a no. divisible by 5.

तब,
$(E_1 \cap E_2)$ = event of getting a no. divisible by both 3 and 5.

= event of getting a no. divisible by 15.

$\therefore E_1 = \{3, 6, 9, \ldots\ldots, 495, 498\}$

$E_2 = \{5, 10, 15, \ldots\ldots, 495, 500\}$

and

$(E_1 \cap E_2) = \{15, 30, 45, \ldots\ldots, 495\}$

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$\therefore m(E_1) = \frac{498}{3} = 166$,

$m(E_2) = \frac{500}{5} = 100$

and

$P(E_1 \cap E_2) = \frac{495}{15} = 33$

$\therefore P(E_1) = \frac{m(E_1)}{m(S)} = \frac{166}{500} = \frac{83}{250}$,

$P(E_2) = \frac{m(E_2)}{m(S)} = \frac{100}{500} = \frac{1}{5}$

and

$P(E_1 \cap E_2) = \frac{m(E_1 \cap E_2)}{m(S)} = \frac{33}{500}$

$\therefore$ P(the chosen no. divisible by 3 or 5)

$= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \left(\frac{83}{250} + \frac{1}{5} - \frac{33}{500}\right)$

$= \frac{233}{500}$

Hence, the reqd. probability is $\frac{233}{500}$

Q. A card is drawn at random from a well shuffled deck of 52 cards. Find the probability of its being a spade or a king.

Ans: मान लिया कि 'S' be the sample space

इसलिए $m(S) = 52$

मान लिया,

E_1 = event of getting a spade
E_2 = event of getting a king.

अब,

$(E_1 \cap E_2)$ = event of getting a king of spades.

clearly, $m(E_1) = 13$, $m(E_2) = 4$. and

$m(E_1 \cap E_2) = 1$.

$\therefore P(E_1) = \frac{m(E_1)}{m(S)} = \frac{13}{52} = \frac{1}{4}$,

$P(E_2) = \frac{m(E_2)}{m(S)} = \frac{4}{52} = \frac{1}{13}$.

and

$P(E_1 \cap E_2) = \frac{m(E_1 \cap E_2)}{m(S)} = \frac{1}{52}$

(P.T.O.)

$\therefore$ P(getting a spade or a king)

$= P(E_1 \text{ or } E_2)$

$= P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

[by the addition theorem for two events]

$= \left(\frac{1}{4} + \frac{1}{13} - \frac{1}{52}\right) = \frac{16}{52} = \frac{4}{13}$.

Hence, the required probability

$= \frac{4}{13}$

Q. Two cards are drawn at random from a well-shuffled pack of 52 cards. What is the probability that either both are red or both are kings?

Ans: मान लिया कि S be the sample space तब,

$n(S)$ = 52 cards में से 2 card को draw करना।

$= {}^{52}C_2$

मान लिया,

E_1 = event that both are red cards

& E_2 = event that both are kings.

Let, $(E_1 \cap E_2)$ = event of getting 2 red Kings.

$m(E_1)$ = no. of ways of drawing 2 red cards out of 26 red cards

$= {}^{26}C_2$.

$m(E_2)$ = no. of ways of drawing 2 Kings out of 4 Kings.

$= {}^{4}C_2$.

$\therefore m(E_1 \cap E_2)$ = no. of ways of drawing 2 red Kings out of 2 red Kings.

$= {}^{2}C_2 = 1$.

$\therefore P(E_1) = \dfrac{m(E_1)}{m(S)} = \dfrac{{}^{26}C_2}{{}^{52}C_2}$;

$P(E_2) = \dfrac{m(E_2)}{m(S)} = \dfrac{{}^{4}C_2}{{}^{52}C_2}$

and,

$P(E_1 \cap E_2) = \dfrac{m(E_1 \cap E_2)}{m(S)} = \dfrac{1}{{}^{52}C_2}$

$\therefore$ P(drawing both red cards or both Kings)

$= P(E_1 \text{ or } E_2)$

$= P(E_1 \cup E_2)$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \left(\frac{^{26}C_2}{^{52}C_2} + \frac{^{4}C_2}{^{52}C_2} - \frac{1}{^{52}C_2} \right)$$

$$= \frac{(^{26}C_2 + {^4C_2} - 1)}{^{52}C_2}$$

$$= \frac{325 + 6 - 1}{1326} = \frac{330}{1326}$$

$$= \frac{55}{221}$$

इसलिए,

required probability $= \frac{55}{221}$

Q. A box contains 100 bolts and 50 nuts. It is given that 50% bolts and 50% nuts are rusted. Two objects are selected from the box at random. Find the probability that either both are bolts or both are rusted.

Ans: Total no. of objects $= (100 + 50)$
$= 150$

माना कि 'S' sample space है। तब,

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$n(S)$ = no. of ways of selecting 2 objects out of 150

$= {}^{150}C_2$

No. of rusted objects.

$= (50\% \text{ of } 100) + (50\% \text{ of } 50)$

$= 50 + 25$

$= 75.$

मान लिया कि

E_1 = event of selecting 2 bolts out of 100 bolts.

E_2 = event of selecting 2 rusted objects out of 75 rusted objects.

$\therefore (E_1 \cap E_2)$ = event of selecting 2 rusted bolts out of 50 rusted bolts

$\therefore n(E_1)$ = number of ways of selecting 2 bolts out of 100

$= {}^{100}C_2.$

$\therefore n(E_2)$ = number of ways of selecting 2 rusted objects out of 75

$= {}^{75}C_2.$

$\therefore n(E_1 \cap E_2)$ = no. of ways of selecting 2 rusted bolts out of 50.

$^{50}C_2$

$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{^{100}C_2}{^{150}C_2}$,

$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{^{75}C_2}{^{150}C_2}$

and,

$P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)}$

$= \frac{^{50}C_2}{^{150}C_2}$

P(selecting both bolts or both rusted objs)

$= P(E_1 \text{ or } E_2)$

$= P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{^{100}C_2}{^{150}C_2} + \frac{^{75}C_2}{^{150}C_2} - \frac{^{50}C_2}{^{150}C_2}$

$= \frac{^{100}C_2 + ^{75}C_2 - ^{50}C_2}{^{150}C_2}$

$= \frac{4950 + 2775 - 1225}{11175}$

$= \frac{6500}{11175} = \frac{260}{447} = 0.58$

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Hence, the required probability = 0.58

Q. If E_1 and E_2 are two events such that $P(E_1) = 0.5$, $P(E_2) = 0.3$ and $P(E_1 \text{ and } E_2) = 0.1$, find?

(i.) $P(E_1 \text{ or } E_2)$
(ii.) $P(E_1 \text{ but not } E_2)$
(iii.) $P(E_2 \text{ but not } E_1)$
(iv.) $P(\text{neither } E_1 \text{ nor } E_2)$

Ans: हमारे पास है:

$P(E_1) = 0.5$, $P(E_2) = 0.3$
$P(E_1 \cap E_2) = 0.1$.

$\therefore P(\overline{E_1}) = 1 - P(E_1)$
$= 1 - 0.5$
$= 0.5$

and,

$P(\overline{E_2}) = 1 - P(E_2)$
$= 1 - 0.3$
$= 0.7$

अब

(i) $P(E_1 \text{ or } E_2)$

$= P(E_1 \cup E_2)$
$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$
$= 0.5 + 0.3 - 0.1$
$= 0.7$

(ii) $P(E_1$ but not $E_2)$

$= P(E_1 \cap \bar{E_2})$

$= P(E_1) - P(E_1 \cap E_2)$

$= 0.5 - 0.1$

$= 0.4$

(iii) $P(E_2$ but not $E_1)$

$= P(E_2 \cap \bar{E_1})$

$= P(E_2) - P(E_2 \cap E_1)$

$= 0.3 - 0.1$

$= 0.2$

(iv) $P($neither E_1 nor $E_2)$

$= P($not E_1 and not $E_2)$

$= P(\bar{E_1} \cap \bar{E_2})$

$= P(\overline{E_1 \cup E_2})$

$= 1 - P(E_1 \cup E_2)$

$= 1 - P(E_1$ or $E_2)$

$= 1 - 0.7$

$= 0.3$

Q. The probability that at least one of the events E_1 and E_2 occurs is 0.6. If the probability of the simultaneous occurrence of E_1 and E_2 is 0.2, find $P(\overline{E_1}) + P(\overline{E_2})$

Ans: दिया हुआ है कि

$$P(E_1 \cup E_2) = 0.6 \text{ and}$$
$$P(E_1 \cap E_2) = 0.2$$

$$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$\Rightarrow P(E_1 \cup E_2) + P(E_1 \cap E_2) = P(E_1) + P(E_2)$$

$$\Rightarrow 0.6 + 0.2 = P(E_1) + P(E_2)$$

$$\Rightarrow P(E_1) + P(E_2) = 0.8$$

$$\Rightarrow \{1 - P(\overline{E_1})\} + \{1 - P(\overline{E_2})\} = 0.8$$

$$\Rightarrow P(\overline{E_1}) + P(\overline{E_2}) = 1.2$$

Q. The probabilities of the occurrences of two events E_1 and E_2 are 0.25 and 0.50 respectively. The probability of their simultaneous occurrence is 0.14. Find the probability that neither E_1 nor E_2 occurs.

~~यहाँ~~ दिया हुआ है कि

$P(E_1) = 0.25$

$P(E_2) = 0.50$

and. $P(E_1 \text{ and } E_2) = P(E_1 \cap E_2)$

$= 0.14$

अब

$P(\overline{E_1}) = 1 - P(E_1)$

$= 1 - 0.25$

$= 0.75$

$P(\overline{E_2}) = 1 - P(E_2)$

$= 1 - 0.50$

$= 0.50$

$\therefore P(\text{neither } E_1 \text{ nor } E_2)$

$= P(\text{not } E_1 \text{ and not } E_2)$

$= P(\overline{E_1} \cap \overline{E_2})$

$= P(\overline{E_1 \cup E_2})$

$= 1 - P(E_1 \cup E_2)$

$= 1 - \{P(E_1) + P(E_2) - P(E_1 \cap E_2)\}$

$= 1 - (0.25 + 0.50 - 0.14)$

$= 1 - 0.61$
$= 0.39$

Hence, the required probability is 0.39.

Q. A card is drawn from a deck of 52 cards. Find the probability of getting a King or a heart or a red card.

Ans: मान लिया कि 'S' sample space है।
अतः, $n(S) = 52$

मान लिए कि E_1, E_2 & E_3 be the events of getting a King, a heart and a red card respectively. Then,

$n(E_1) = 4$
$n(E_2) = 13$
$n(E_3) = 26$

$(E_1 \cap E_2)$ = event of getting a King of hearts,
$(E_2 \cap E_3)$ = event of getting a heart
$(E_3 \cap E_1)$ = event of getting a red King.
[∵ a heart is a red card also;]

and $(E_1 \cap E_2 \cap E_3)$ = event of getting a King of hearts.

$\therefore m(E_1 \cap E_2) = 1$

$m(E_2 \cap E_3) = 13$

$m(E_3 \cap E_1) = 2$

and

$m(E_1 \cap E_2 \cap E_3) = 1$

$\therefore P(E_1) = \frac{m(E_1)}{m(S)} = \frac{4}{52} = \frac{1}{13};$

$P(E_2) = \frac{m(E_2)}{m(S)} = \frac{13}{52} = \frac{1}{4};$

$P(E_3) = \frac{m(E_3)}{m(S)} = \frac{26}{52} = \frac{1}{2};$

$P(E_1 \cap E_2) = \frac{m(E_1 \cap E_2)}{m(S)} = \frac{1}{52};$

$P(E_2 \cap E_3) = \frac{m(E_2 \cap E_3)}{m(S)} = \frac{13}{52} = \frac{1}{4},$

$P(E_3 \cap E_1) = \frac{m(E_3 \cap E_1)}{m(S)} = \frac{2}{52} = \frac{1}{26}$

&

$P(E_1 \cap E_2 \cap E_3) = \frac{m(E_1 \cap E_2 \cap E_3)}{m(S)} = \frac{1}{52}$

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$\therefore$ P(getting a King or a heart or a red card)

$= P(E_1 \text{ or } E_2 \text{ or } E_3)$

$= P(E_1 \cup E_2 \cup E_3)$

$= P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_3 \cap E_1) + P(E_1 \cap E_2 \cap E_3)$

$= \left(\frac{1}{13} + \frac{1}{4} + \frac{1}{2} - \frac{1}{52} - \frac{1}{4} - \frac{1}{26} + \frac{1}{52}\right)$

$= \frac{28}{52} = \frac{7}{13}.$

Hence, the required probability is $\frac{7}{13}$.

Q. If A and B are two events associated with a random experiment for which $P(A) = 0.60$, $P(A \text{ or } B) = 0.85$ and $P(A \text{ and } B) = 0.42$, find $P(B)$

Ans: given

$P(A) = 0.60$

$P(A \text{ or } B) = 0.85$

$P(A \text{ and } B) = 0.42$

हमको निकालना है P(B), इसलिए formulae लगाएँ :-

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Values को रखने पर मिला :-

$0.85 = 0.60 + P(B) - 0.42$
$0.85 = 0.18 + P(B)$
$0.85 - 0.18 = P(B)$
$P(B) = 0.67$

$\therefore P(B) = 0.67$

Q. Let A and B be two events associated with a random experiment for which $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \text{ or } B) = 0.6$, find $P(A \text{ and } B)$.

Ans: दिया हुआ है कि

$P(A) = 0.4$
$P(A \text{ or } B) = 0.6$
$P(B) = 0.5$

हमें निकालना है $P(A \text{ and } B) = ?$

तो हम यहाँ पर formulae लगाएँगे

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\Rightarrow 0.6 = 0.4 + 0.5 - P(A \text{ and } B)$
$\Rightarrow 0.6 = 0.9 - P(A \text{ and } B)$
$\Rightarrow P(A \text{ and } B) = 0.9 - 0.6$

$\therefore P(A \text{ and } B) = 0.3$

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Q. In a random experiment, let A and B be events such that P(A or B) = 0.7, P(A and B) = 0.3 and $P(\bar{A}) = 0.4$. Find P(B).

Ans: दिया हुआ है कि

$$P(\bar{A}) = 0.4$$
$$P(A \text{ or } B) = 0.7$$
$$P(A \text{ and } B) = 0.3$$

हमें निकालना है कि P(B) क्या होगा

$\because P(\bar{A}) = 0.4$

$$P(A) = 1 - P(\bar{A}) \quad \left\{ \because P(A) = 1 - P(\bar{A}) \right\}$$
$$= 1 - 0.4$$
$$= 0.6$$

Using the formulae :—

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$\Rightarrow 0.7 = 0.6 + P(B) - 0.3$

$\Rightarrow 0.7 = 0.3 + P(B)$

$\Rightarrow P(B) = 0.4$

$\therefore P(B) = 0.4$

(P.T.O.)

Q. If A and B be two events associated with a random experiment such that $P(A) = 0.25$, $P(B) = 0.4$ and $P(A \text{ or } B) = 0.5$, find the values of :-

(i) $P(A \text{ and } B)$

(ii) $P(A \text{ and } \overline{B})$

Ans: (i) दिया हुआ है कि

$P(A) = 0.25$

$P(B) = 0.4$

$P(A \text{ or } B) = 0.5$

हमें निकालना है $P(A \text{ and } B)$

$\because P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

सारा values को रखे तो, मिला

$\Rightarrow 0.5 = 0.25 + 0.4 - P(A \text{ and } B)$

$\Rightarrow 0.5 = 0.65 - P(A \text{ and } B)$

$\Rightarrow P(A \text{ and } B) = 0.65 - 0.5$

$\Rightarrow P(A \text{ and } B) = 0.15$

$\Rightarrow P(A \text{ and } B) = 0.15.$

(P.T.O.)

(ii) $P(A) = 0.25$, $P(A \text{ and } B) = 0.15$

तो हमें निकालना है कि $P(A \text{ and } \bar{B})$

$\because P(A \text{ and } \bar{B}) = P(A) - P(A \text{ and } B)$

$P(A \text{ and } \bar{B}) = 0.25 - 0.15$

$= 0.10$

Q. If A and B be two events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$, find

(i) $P(\bar{A} \cap B)$ (ii) $P(A \cap \bar{B})$

Ans (i) दिया हुआ है कि :—

$P(A) = 0.3$

$P(B) = 0.2$

$P(A \cap B) = 0.1$

हमको निकालना है $P(\bar{A} \cap B)$

$\because P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$\Rightarrow P(\bar{A} \cap B) = 0.2 - 0.1$

$\Rightarrow P(\bar{A} \cap B) = 0.1$

(P.T.O.)

(ii.) दिया हुआ है कि :-

$P(A) = 0.3$
$P(B) = 0.2$
$P(A \cap B) = 0.1$

हमें निकालना है $P(A \cap \bar{B})$

$\because P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$\Rightarrow P(A \cap \bar{B}) = 0.3 - 0.1$

$\Rightarrow P(A \cap \bar{B}) = 0.2$

Q. If A and B are two mutually exclusive events such that $P(A) = 1/2$ and $P(B) = 1/3$, find $P(A \text{ or } B)$.

Ans: दिया हुआ है कि A और B mutually exclusive events है।

$P(A) = \frac{1}{2}$ $P(B) = \frac{1}{3}$

हमे निकालना है $P(A \text{ or } B)$

$\because P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

हमको पता है कि Mutually ~~exclusivents~~ exclusive events के लिए $P(A \text{ and } B) = 0$ होता है।

$$P(A \text{ or } B) = \frac{1}{2} + \frac{1}{3}$$

$$P(A \text{ or } B) = \frac{3+2}{6} = \frac{5}{6}$$

Q. Let A and B be two mutually exclusive events of a random experiment such that P(not A) = 0.65 and P(A or B) = 0.65, find P(B).

Ans: दिया हुआ है कि

A और B दो mutually exclusive events.

$P(\text{not } A) = P(\bar{A}) = 0.65$

$P(A \text{ or } B) = 0.65$

अब हमें निकालना है P(B)

$\because P(A) = 1 - P(\bar{A})$

$P(A) = 1 - 0.65$

$P(A) = 0.35$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.65 = 0.35 + P(B) - P(A \cap B)$

For Mutually exl. events A & B ; $(A \cap B) = 0$

$\Rightarrow 0.65 = 0.35 + P(B)$

$\Rightarrow P(B) = 0.65 - 0.35$

$= 0.30$

$\therefore P(B) = 0.30$

Q. A, B, C are three mutually exclusive and exhaustive events associated with a random experiment. If P(B) = 3/2 , P(A) and C) = P(B) = (3/2) · P(A) and P(C) = (1/2) · P(B), find P(A).

Ans: दिया हुआ है कि A, B, C mutually exclusive events & exhaustive events.

$P(B) = (3/2) \cdot P(A)$
$P(C) = (1/2) \cdot P(B)$

हमें निकालना है कि P(A) क्या होगा

Mutually exclusive events A, B & C के लिए

$$P(A \text{ and } B) = P(B \text{ and } C) = P(A \text{ and } C) = 0$$

मान लिया कि $P(A) = x$

Now,

$$P(B) = \frac{3}{2} \cdot x \quad \Big\} \quad P(C) = \frac{1}{2} \times \frac{3}{2} x$$
$$= \frac{3}{4} x$$

Since,

$$P(A) + P(B) + P(C) = 1$$

(P.T.O.)

putting values, we get :

$\Rightarrow x + \frac{3}{2}x + \frac{1}{2} \times \frac{3}{2} \times x = 1$

$\Rightarrow x + \frac{3}{2}x + \frac{3}{4}x = 1$

$\Rightarrow \frac{4x + 6x + 3x}{4} = 1$

$\Rightarrow 13x = 4$

$\Rightarrow x = \frac{4}{13}$ इसलिए, $P(A) = \frac{4}{13}$

Q The probability that a company executive will travel by plane is (2/5) and that he will travel by train is (1/3). Find the probability of his travelling by plane or train.

Ans:

मान लिया कि
'A' denote the event that a company executive will travel by plane.

'B' denote the event that he travels by train.

दिया हुआ है कि

$P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$

हमे निकालना है probability of a company executive will be travelling by plane or train

$= P(A \text{ or } B)$

$\because P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

तो समझो यहाँ की

probability of a company executive will be travelling in both plane and train.

$= P(A \text{ and } B) = 0$

क्योंकी वह plane और train पर एक समय पर सफर नहीं कर सकता है!

$P(A \text{ or } B) = \frac{2}{5} + \frac{1}{3} - 0$

$= \frac{6+5}{15} = \frac{11}{15}$

Hence,

Probability of a company executive will be travelling by plane or train

$= P(A \text{ or } B) = \frac{11}{15}$

Q. From a well shuffled pack of 52 cards, a card is drawn at random. Find the probability of its being a king or queen.

Ans: माना लिया कि,
A denote the event that card drawn is king.
B denote the event that card drawn is queen.

एक pack of 52 cards में 4 kings एवं 4 queen cards होते हैं।

दिया हुआ है कि

$$P(A) = \frac{4}{52} \qquad P(B) = \frac{4}{52}$$

हमें निकालना है कि
probability that card drawn is king or queen
$= P(A \text{ or } B)$

$\because$ Probability $= \dfrac{\text{favourable no. of outcomes}}{\text{total no. of outcomes}}$

क्योंकि

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$P(A) = \frac{4}{52}$ { क्योंकि favourable no. of outcomes = 4 & total no. of outcomes = 52 }

$P(B) = \frac{4}{52}$ { क्योंकि favourable no. of outc. = 4 & total no. of outcomes = 52 }

तो,

probability that card drawn is King or queen = P(A and B) = 0

क्योंकि एक card same time पर King और queen दोनों नहीं हो सकते

$$P(A \text{ or } B) = \frac{4}{52} + \frac{4}{52} - 0$$

$$P(A \text{ or } B) = \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$$

$$P(A \text{ or } B) = \frac{2}{13}$$

∴ Probability of a card drawn is King or queen

$$= P(A \text{ or } B) = \frac{2}{13}$$

Q. From a well-shuffled pack of cards, a card is drawn at random. Find the probability of its being either a queen or a heart.

(P.T.O.)

Ans: माना लिया कि
A denote the event that the card drawn is queen. &

B denote the event that the card drawn is heart.

एक pack of 52 cards में 4 queen cards होते हैं एवं 13 heart cards होते हैं।

दिया हुआ है कि

$$P(A) = \frac{4}{52}, \quad P(B) = \frac{13}{52}$$

हमें निकालना है कि Probability that card drawn is either a queen or heart.

$= P(A \text{ or } B)$

{ ∵ probability = $\frac{\text{favourable no. of outcomes}}{\text{total no. of outcomes}}$ }

इसलिए,

$P(A) = \frac{4}{52}$ { क्योंकि favourable no. of outcomes = 4 & total no. of outcomes = 52 }

$P(B) = \frac{13}{52}$ { क्योंकि favourable no. of outcomes = 13 & total no. of outcomes = 52 }

Probability that card drawn is both queen and heart

$= P(A \text{ and } B) = \frac{1}{52}$

{क्योंकि यहाँ सिर्फ एक ही कार्ड ऐसा है जो queen & heart दोनों है यानी queen of hearts}

$$P(A \text{ or } B) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$P(A \text{ or } B) = \frac{4+13-1}{52} = \frac{16}{52} = \frac{4}{13}$$

Hence,

Probability of a card drawn is either a queen or heart

$= P(A \text{ or } B)$

$= \frac{4}{13}$

Q. A card is drawn at random from a well-defined deck of 52 cards. Find the probability of its being a spade or a king.

(P.T.O.)

Ans: मान लिया कि
A denote the event that the card drawn is spade.

B denote the event that card drawn is King.

एक 52 कार्ड्स के pack में 13 spade cards और 4 King cards होते हैं

दिया हुआ है कि :—

$P(A) = \frac{13}{52}$, $P(B) = \frac{4}{52}$

हमें निकालना है कि Probability that card drawn is either a queen or heart.
$= P(A \text{ or } B)$

{ ∵ $\text{Probability} = \frac{\text{favourable no. of outcomes}}{\text{total no. of outcomes}}$ }

$P(A) = \frac{13}{52}$ { as favourable no. of outcomes = 13 & total no. of outcomes = 52 }

$P(B) = \frac{4}{52}$ { as favourable no. of outcomes = 4 & total no. of outcomes = 52 }

The probability that card is drawn is both spade and king

$= P(A \text{ and } B)$

$= 1$

{क्योंकी एक card है जिसमे both Spade & King i.e. King of Spades}

$\because P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\Rightarrow P(A \text{ or } B) = \frac{13}{52} + \frac{4}{52} - 1$

$\Rightarrow P(A \text{ or } B) = \frac{16}{52} = \frac{4}{13}$

Here,

Probability of a card drawn is either a spade or king

$= P(A \text{ or } B)$

$= \frac{4}{13}$

Q. A number is chosen from the numbers 1 to 100. Find the probability of its being divisible by 4 or 6.

Ans: माना लिया

A denote the event that the no. is divisible by 4.

B denote the event that the no. is divisible by 4.

हमें निकालना है probability that the number is both divisible by 4 or 6

= P(A or B)

∵ probability = $\frac{\text{favourable no. of outcomes}}{\text{total no. of outcomes}}$

~~P(A or B) = P~~

Numbers from 1 to 100 divisible by 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100

1 से 100 के बीच 25 ऐसे no.s हैं जो 4 से divisible होंगे।

1 से 100 के बीच 16 no.s होंगे जो 6 से divisible होंगे।

favourable no. of outcomes = 16

इसलिए,

favourable no. of outcomes = 16

Total no. of outcomes = 100
क्योंकि 1 से 100 तक 100 नंबर्स है।

$$P(B) = \frac{16}{100}$$

Numbers from 1 to 100 divisible by both 4 and 6 are 12, 24, 36, 48, 60, 72, 84, 96.

1 से 100 के बीच 8 numbers है जो 4 & 6 दोनों से divisible होगा।

Favourable no. of outcomes = 8

$$P(A \text{ and } B) = \frac{8}{100}$$

$$\because P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{25}{100} + \frac{16}{100} - \frac{8}{100}$$

$$= \frac{25+16-8}{100} = \frac{33}{100}$$

The probability that the no. is both divisible by 4 or 6

$= P(A \text{ or } B) = \frac{33}{100}$

Q A die is thrown twice. What is the probability that at least one of the two throws comes up with the number 4?

Ans: दिया गया है कि एक dice को फेंके

हमें find करना है कि probability that at least one of the two throws comes up with the number 4

$\therefore \text{probability} = \frac{\text{favourable outcomes}}{\text{total outcomes}}$

A die is numbered from 1 to 6.

जब एक die को दो बार throw किए तो,

total no. of outcomes $= 6^2 = 36$

Favourable outcomes =

{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (1,4), (2,4), (3,4), (5,4), (6,4)}

Favourable no. of outcomes = 11

Probability that at least one of the two throws comes up with the number 4

$= \frac{11}{36}$.

The probability that at least one of the two throws comes up with the number 4

$= \frac{11}{36}$.

Q. Two dice are tossed once. Find the probability of getting an even no. on the first die or a total of 8.

Ans: दिया गया है कि दो dice को एक बार toss किया।

हमे निकालना है Probability of getting an even number on the first die or a total 8.

A die is numbered from 1 to 6

जब दो dice को एक बार toss करेंगे तो,

total no. of outcomes $= 6^2 = 36$

मान लिया

A denote the event of getting an even number on the first die (a) &

B denote the event of getting a total of 8

For getting an even no. on the first die

favourable outcomes =

$\{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

Favourable no. of outcomes = 18

Probability of getting an even no. on the first die

$= P(A) = \frac{18}{36}$

For getting a total of 8:

Favourable outcomes =

$\{(2,6), (4,4), (6,2), (5,3), (3,5)\}$

favourable outcomes = 5

Probability of getting a total of 8

$= P(A) = \frac{5}{36}$

For getting an even number on the first die and a total of 8:

favourable outcomes

$= \{(2,6), (4,4), (6,2)\}$

Probability of getting an even number on the first die and a total of 8

$= P(A \text{ and } B)$

$= \frac{3}{36}$

$\because P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ or } B) = \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$

$P(A \text{ or } B) = \frac{18+5-3}{36} = \frac{20}{36} = \frac{5}{9}$

Probability of getting an even no. on the first die or a total 8

$= \frac{5}{9}$

Q. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither divisible by 3 nor by 4.

Ans: दिया गया है कि दो dice को throw किए

Sample Space:

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

तो, इसको निकालना है

P(sum of faces neither divisible by 3 nor by 4)

Sum = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

Sum neither divisible by 3 nor 4

= {2, 3, 5, 7, 10, 11}

$$P = \frac{\text{no. of favourable outcomes}}{\text{total outcomes}}$$

P(sum of faces neither divisible by 3 nor by 4)

$$= \frac{5}{11}$$

$\therefore$ probability $= \frac{5}{11}$.

Q. In a class, 30% of the students offered mathematics 20% offered chemistry and 10% offered both. If a student is selected at random, find the probability that he has offered mathematics or chemistry.

Ans: दिया गया है :—

Math Students = 30%
Chemistry Students = 20%

Math & Chemistry both = 10%

हमको निकालना है, P(Math or Chemistry)

अत

$P(\text{math}) = 30\% = \frac{30}{100} = 0.30$

$P(\text{Chemistry}) = 20\% = \frac{20}{100} = 0.20$

$P(\text{Math} \cap \text{Chemistry}) = 10\% = \frac{10}{100} = 0.10$

We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(\text{Math} \cup \text{Chemistry})$

$= 0.30 + 0.20 - 0.10$

$= 0.40$

Hence,

number of students studying math or chemistry are 40%.

Q. The probability that Hemant passes in English is (2/3), and the probability that he passes in Hindi is (5/9). If the probability of his passing both the subjects is (2/5), find the probability that he will pass in at least one of these subjects.

Ans: माना लिया कि :-

'A' → denotes the event that Hemant passes in english

'B' → denotes the event that Hemant passes in hindi

दिया हुआ कि है :-

$P(A) = \frac{2}{3}$, $P(B) = \frac{5}{9}$

$P(A \text{ and } B) = \frac{2}{5}$

इसको निकालना है;

probability that he will pass in at least one of these subjects

$= P(A \text{ or } B)$

$$\left\{ \because P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \right\}$$

$$P(A \text{ or } B) = \frac{2}{3} + \frac{5}{9} - \frac{2}{5}$$

$$P(A \text{ or } B) = \frac{30 + 25 - 18}{45} = \frac{37}{45}$$

$$P(A \text{ or } B) = \frac{37}{45}.$$

Hence,

The probability that she will pass in at least one of these subjects

$$= P(A \text{ or } B)$$

$$= \frac{37}{45}.$$

Q. The probability that a person will get an electrification contract is a (2/5) and the probability that he will not get a plumbing contract is (4/7). If the probability of getting at least one contract is (2/3), what is the probability that he will get both?

(P.T.O.)

Ans: मान लिया किया

'A' denote the event that a person will get electrification contract.

'B' denote the event that a person will get a plumbing contract.

दिया गया है कि :-

$P(A) = \frac{2}{5}$, $P(\text{not } B) = P(\bar{B}) = \frac{4}{7}$

$P(A \text{ or } B) = \frac{2}{3}$.

हमें निकालना है;

Probability that he will get both electrification and plumbing contract
$= P(A \text{ and } B)$

$\because P(B) = 1 - P(\bar{B})$

$P(B) = 1 - \frac{4}{7} = \frac{7-4}{7} = \frac{3}{7}$

$P(A \text{ or } B)$ = probability of getting at least one contract
$= \frac{2}{3}$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{2}{3} = \frac{2}{5} + \frac{3}{7} - P(A \text{ and } B)$

$\Rightarrow P(A \text{ and } B) = \frac{29}{35} - \frac{2}{3}$

$\Rightarrow P(A \text{ and } B) = \frac{17}{105}$

The probability that he will get both electrification and plumbing contract

$= \frac{17}{105}$

Q. The probability that a patient visiting a dentist will have a tooth extracted is 0·06, the probability that he will have a cavity filled is 0·2, and the probability that he will have a tooth extracted or a cavity filled is 0·23. What is the probability that he will have a tooth extracted as well as a cavity filled?

Ans: मान लिया कि

'A' denote the event that a patient visiting a dentist will have a tooth extracted &

"B" denote the event that a patient will have a cavity filled.

दिया गया है कि :-

$P(A) = 0.06$

$P(B) = 0.2$

$P(A \text{ or } B) = 0.23$

हमको निकालना है Probability that he will have a tooth extracted and a cavity filled

$= P(A \text{ and } B)$

Probability that he will have a tooth extracted or a cavity filled

$= P(A \text{ or } B) = 0.23$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.23 = 0.06 + 0.2 - P(A \text{ and } B)$

$\Rightarrow 0.23 = 0.26 - P(A \text{ and } B)$

$\Rightarrow P(A \text{ and } B) = 0.26 - 0.23$

$= 0.03$

इसलिए,

Probability that he will have a tooth extracted and a cavity filled

$= P(A \text{ and } B)$

$= 0.03$

Q. In a town of 6000 people, 1200 are over 50 years old, and 2000 are females. It is known that 30% of the females are over 50 years. What is the probability that a randomly chosen individual from the town is either female or over 50 years?

Ans: मान लिया कि

'A' denote the event that the chosen individual is female (&)

'B' denote the event that the chosen individual is over 50 years old.

दिया गया है कि :-

Town में 6000 people हैं।

1200 50 साल से ज्यादा की उम्र के हैं।

2000 females हैं।

हमको निकालना है :-

Probability that a randomly chosen individual from the town is either female or male over 50 years.

= P(A or B)

$$\because \text{Probability} = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

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For the event A,

There are 2000 females present in a town of 6000 people.

Favourable no. of outcomes = 2000
Total no. of outcomes = 6000

$$P(A) = \frac{\overset{1}{\cancel{2000}}}{\underset{3}{\cancel{6000}}} = \frac{1}{3}$$

For the event B,

There are 1200 are over 50 years of age in a town of 6000 people.

Favourable no. of outcomes = 1200
Total no. of outcomes = 6000

$$P(B) = \frac{\cancel{1200}}{\cancel{6000}} = \frac{1}{5}$$

30% of the females are over 50 years.
For the event A and B,

$$\frac{30}{100} \times 2000 = 600$$ females are over 50 years of age.

(P.T.O.)

Favourable no. of outcomes = 600

$P(A \text{ and } B) = \frac{600}{6000} = \frac{1}{10}$,

$\because P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\Rightarrow P(A \text{ or } B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{10}$

$\Rightarrow P(A \text{ or } B) = \frac{10+6-3}{30} = \frac{13}{30}$

$\Rightarrow P(A \text{ or } B) = \frac{13}{30}$

इसलिए,

The probability that a randomly chosen individual from the town is either female or over 50 years.

$= P(A \text{ or } B)$

$= \frac{13}{30}$.

Conditional Probability

P(A|B) can be said as:
- Probability of occurence of event A when B has already occurred
- Probability of occurence of event A when B is taken as sample space.

अब तक हमने probability का एक formulae पढ़ा था i.e. P(event A) = favourable outcomes to A divided by total no. of outcomes. पर अब हम उसमे एक condition डाल रहे हैं जो उस condn. को हमने ऊपर लिखा है अब समझिएगा कि उन दोनों में slight difference है

तो सबसे पहले समझते हैं इस definition को:-

⇒ P(A|B) = Probability of occurence of event A when event B has already occured.

इसके लिए example लिए :-

एक bag में 4 black balls & 5 red balls हैं तो जो पहला ball draw किए वह red है, क्या probability है कि दूसरा ball is black?

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तो इस question का solution कुछ ऐसा होगा:-

Bag

let, A = 4 black balls है
B = 5 Red balls है

तो जब पहला event i.e. drawing a red ball हुआ तो कुल 9 balls मेरे पास थे जिसमें से सिर्फ 8 बचे

इसलिए, probability that second ball drawn is black.

$= \frac{4}{8}$ → Favourable / → Total.

क्योंकी एक ball निकाल चुके थे इस event से पहले इसलिए अब sample space में 8 balls होंगी।

⇒ अब हम समझते है दूसरा definition को :-

P(A|B) = Probability of occurence of event A when event B is taken as sample space.

(P.T.O.)

इसके लिए example लिए :-

हमने 2 dice को roll किया

तो हमने event ऐसे define किए :-

A = sum is 8

B = even number on first dice

अब देखो

A event के लिए possibilities :-

{ (6,2), (5,3), (4,4)
(3,5), (2,6) }

B event के लिए possibilities :-

{ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

अब देखो इस experiment के लिए जो original sample space बनेगा उसमें 36 elements होगें।

तो अब देखो :-

P(A|B) = probability of A when B is taken as sample space.

तो अब sample space event B को लेंगे यानी :-

$$S = \{ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$$
$$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$$
$$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6) \}$$

18 outcomes

favourable outcome for event A

$$= \{ (6,2),\ (5,3),\ (4,4),\ (3,5),\ (2,6) \}$$

इसलिए

$$P(A|B) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{3}{18} = \frac{1}{6}$$

(P.T.O.)

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अब हमने जब theoretically conditional probability के concept को समझ लिया है। तो आइए अब इसको formulae समझे :-

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \; ; \; P(B) \neq 0$$

(or),

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \; ; \; P(A) \neq 0$$

Q. Let A and B be the events such that

$P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$.

find :-

(i.) $P(A|B)$ (iii.) $P(A \cup B)$

(ii.) $P(B|A)$ (iv.) $P(\bar{B}|\bar{A})$

Ans: दिया गया है।

A और B events है such that

$P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ & $P(A \cap B) = \frac{4}{13}$

(i.) $P(A|B)$

$$= \frac{P(A \cap B)}{P(B)} = \frac{4}{13} \times \frac{13}{9} = \frac{4}{9}$$

$\therefore P(A|B) = \frac{4}{9}$.

(ii.) P(B|A)

$$= \frac{P(A \cap B)}{P(A)}$$

$$= \frac{4}{13} \times \frac{13}{7} = \frac{4}{7}$$

$\therefore P(B/A) = \frac{4}{7}$.

(iii.) P(A∪B)

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{7}{13} + \frac{9}{13} - \frac{4}{13}$$

$$= \frac{12}{13}$$

(iv.) $P(\bar{B}/\bar{A})$

$$= \frac{P(\bar{A} \cap \bar{B})}{P(\bar{A})}$$

by using de-morgan's law,
$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

$$= \frac{P(\overline{A \cup B})}{P(\bar{A})}$$

$$= \frac{1 - P(A \cup B)}{1 - P(A)}$$

$$= \frac{1 - \frac{12}{13}}{1 - \frac{7}{13}} = \frac{1}{6}.$$

Q. Let A and B be the events such that $P(A) = \frac{5}{11}$, $P(B) = \frac{6}{11}$ and $P(A \cup B) = \frac{7}{11}$.

find :-

(i.) $P(A \cap B)$ (iii.) $P(B | A)$

(ii.) $P(A | B)$ (iv.) $P(\bar{A} / \bar{B})$

Ans: दिया हुआ है कि :-

A and B be the events such that

$P(A) = \frac{5}{11}$, $P(B) = \frac{6}{11}$ and

$P(A \cup B) = \frac{7}{11}$

(i.) $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\Rightarrow P(A \cap B) = \frac{5}{11} + \frac{6}{11} - \frac{7}{11}$

$= \frac{4}{11}$

(ii.) $P(A | B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{4}{11} \times \frac{11}{6}$

$= \frac{4}{6}$ (cancelled to $\frac{2}{3}$)

$= \frac{2}{3}$

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(iii) $P(B|A)$

$$= \frac{P(A \cap B)}{P(A)} = \frac{4}{\cancel{11}} \times \frac{\cancel{11}}{5} = \frac{4}{5}$$

(iv) $P(\bar{A}|\bar{B})$

$$= \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$$

$$= \frac{P(\overline{A \cup B})}{P(\bar{B})}$$

$$= \frac{1 - P(A \cup B)}{1 - P(B)}$$

$$= \frac{1 - \frac{7}{11}}{1 - \frac{6}{11}}$$

$$= \frac{\frac{11-7}{\cancel{11}}}{\frac{11-6}{\cancel{11}}}$$

$$= \frac{4}{5}$$

Q. Let A and B be the events such that

$P(A) = \frac{3}{10}$, $P(B) = \frac{1}{2}$ and $P(B|A) = \frac{2}{5}$.

Find :-

(i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A|B)$

Ans: दिया गया है कि

A & B be the events, such that

$P(A) = \frac{3}{10}$, $P(B) = \frac{1}{2}$ & $P(B|A) = \frac{2}{5}$

(i) $P(A \cap B)$

$\because P(B|A) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow P(A \cap B) = P(B|A) \cdot P(A)$

$\Rightarrow P(A \cap B) = \frac{2}{5} \times \frac{3}{10} = \frac{3}{25}$

(ii) $P(A \cup B)$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = \frac{3}{10} + \frac{1}{2} - \frac{3}{25}$

$\Rightarrow P(A \cup B) = \frac{15 + 25 - 6}{50}$

$= \frac{34}{50}$

(iii.) $P(A|B)$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{3}{25} \times 2 = \frac{6}{25}.$$

Q. let A and B be the events such that :-

$2 \cdot P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$.

find

(i) $P(A \cap B)$ (ii.) $P(A \cup B)$

Ans: दिया गया है कि

A & B be the events

such that

$$2 \cdot P(A) = P(B) = \frac{5}{13} \quad \&$$

$$P(A|B) = \frac{2}{5}.$$

(i) $P(A \cap B)$

$$\because P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(A|B) \cdot P(B)$$

$$\Rightarrow P(A \cap B) = \frac{2}{5} \times \frac{5}{13}$$

$$= \frac{2}{13}$$

(ii) P(AUB)

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$

$= \frac{5 + 10 - 4}{26} = \frac{11}{26}$

Q. A die is rolled. If the outcomes is an even number, what is the probability that it is a number greater than 2?

Ans: एक die में 6 faces होते हैं और इसका sample space = {1, 2, 3, 4, 5, 6}

Total no. of outcomes = 6

मान लिया कि P(A) probability है for getting an even number.

The sample space of A = {2, 4, 6}

$\therefore P(A) = \frac{3}{6} = \frac{1}{2}$

मान लिया कि P(B) probability है for getting a number whose value is greater than 2.

The sample space of B = {3, 4, 5, 6}

$\therefore (A \cap B) = \{4, 6\}$

$\therefore P(A \cap B) = \frac{2}{6} = \frac{1}{3}.$

by conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

जहाँ, P(A|B) is the probability of occurrence of the event A given that B event has already occurred.

The probability of getting a number greater than 2 given that the outcome is even is given by :-

$P(B|A)$

$$= \frac{P(A \cap B)}{P(A)}$$

$$= \frac{1/3}{1/2} = \frac{2}{3}.$$

(P.T.O.)

Q. A coin is tossed twice If the outcome is atmost one tail, what is the probability that both head and tail have appeared?

Ans: एक Coin के पास 2 sides होते हैं और sample space $S = \{H, T\}$

The total no. of outcomes = 2

~~एक coin को दो बार उछाला~~

माना लिया, P(A) be the probability of getting at most 1 tail.

तो sample space of $A = \{(H, H), (H, T), (T, H)\}$

माना लिया, P(B) be the probability of getting a head.

तो sample space of $B = \{H\}$.

$\therefore P(B) = \frac{1}{2}$.

अब देखो probability of getting at most one tail and a head

i.e. $P(A \cap B) = \{(H, H)\}$

$\therefore P(A \cap B) = \frac{1}{3}$

P(A|B) → probability of occurrence of the event A given that B has already occured.

The probability that both head and tail have appeared:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/3}{1/2} = \frac{2}{3}$$

Q. Three coins are tossed simultaneously. Find the probability that all coins show heads if at least one of the coin shows a head.

Ans: जब तीन coins को एक साथ toss किए तो total no. of outcomes $= 2^3 = 8$

और

sample space होगा

S = { (H, H, H), (H, H, T), (H, T, T), (H, T, H), (T, H, T), (T, T, H), (T, H, H), (T, T, T) }

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माना किया P(A) be the probability of getting 3 heads.

The sample space of A = {(H, H, H)}

$$\therefore P(A) = \frac{1}{8}$$

माना किया P(B) be the probability of getting at least 1 head.

Probability of one head = 1 − probability of no heads

$$= 1 - \frac{1}{8} = \frac{7}{8}$$

$$\therefore P(B) = \frac{7}{8}$$

The probability that the throw is either all heads or at least one head

i.e.

$$P(A \cup B) = \frac{7}{8}$$

अब,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\Rightarrow P(A \cap B) = \frac{1}{8} + \frac{7}{8} - \frac{7}{8}$$

$$= \frac{1}{8}$$

P(A|B) = The probability that all coins show heads if at least one of the coins showed a head.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/8}{7/8} = \frac{1}{7}$$

Q. Two unbaised dice are thrown. Find the probability that the sum of the numbers appearing is 8 or greater, if 4 appears on the first die.

Ans: दो die हैं हर एक के पास 6 faces हैं उनको एक साथ उछाला। तो मेरे पास total no. of outcomes $= 6^2 = 36$.

मान लिया

P(A) be the probability of getting a sum greater than 8.

P(B) be the probability of getting 4 on the first die.

B के लिए sample space

$= \{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}$

$\therefore P(B) = \frac{6}{36} = \frac{1}{6}$

माना लिया कि $P(A \cap B)$ है probability of getting 4 on the first die and the sum greater than or equal to 8.

तो $(A \cap B)$ के लिए sample space होगा :-

$\{(4,4), (4,5), (4,6)\}$

$\therefore P(A \cap B) = \frac{3}{36} = \frac{1}{12}$

Now,

$P(A|B)$ = probability that sum of the no.s is greater than or equal to 8 given that 4 was thrown first.

$P(A|B)$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{12}}{\frac{1}{6}}$$

$$= \frac{1}{2}$$

(P.T.O.)

Q. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared at least once?

Ans: एक dice जिसको दो बार फेंके उसके पास total outcomes होंगे $6^2 = 36$

माना लिया,

P(A) be the probability of getting the number 5 at least once.

P(B) be the probability of getting sum of 8.

The sample space of B

$= \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$\therefore P(B) = \frac{5}{36}$

माना लिया कि $P(A \cap B)$ be the probability of getting the no. 5 at least once and the sum equal to 8.

The sample space of $(A \cap B)$

$= \{(3,5), (5,3)\}$

$\therefore P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

using conditional probability

P(A|B) = The probability that the no. 5 have appeared at least once given that the sum is 8.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/18}{5/36} = \frac{\cancel{36}^{2}}{\cancel{18} \times 5} = \frac{2}{5}$$

Q. Two dice were thrown and it is known that the numbers which come up were different. Find the probability that the sum of the two numbers was 5.

Ans: दो die जिसके पास 6 faces हैं हर एक dice पर उसको हम एक साथ उछाले तो total outcomes होंगे $6^2 = 36$

मान लिया

P(A) be the probability of getting a sum equal to 5.

P(B) be the probability of getting 2 different numbers.

Probability of getting 2 different numbers

$= 1 -$ probability of getting same no.s

$= 1 - \frac{6}{36}$

$\Rightarrow 1 - \frac{1}{6} = \frac{5}{6}$

$\therefore P(B) = \frac{5}{6}$

मान लिया कि $P(A \cap B)$ be the probability of getting a sum = 5 (&) two different numbers at the same time.

The sample space of $(A \cap B)$

$= \{(1,4), (2,3), (3,2), (4,1)\}$

$\therefore P(A \cap B) = \frac{4}{36} = \frac{1}{9}$

$P(A|B)$ = The probability that the sum = 5 given that two different no.s were thrown.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/9}{5/6} = \frac{2}{15}$$

Q. A coin is tossed and then a die is thrown. Find the probability of obtaining a 6, given that a head came up.

Ans: एक coin को toss किए उसके बाद एक die को throw किए।
एक coin जिसमें 2 sides होते हैं उसमें total 2 outcomes होते हैं viz. {H, T}
एक die में 6 faces हैं इसलिए उसमें total 6 outcomes होंगे i.e. {1, 2, 3, 4, 5, 6}

मान लिया P(A) be the probability of getting the number 6.

$$\therefore P(A) = \frac{1}{6}$$

मान लिया P(B) be the probability of getting a head.

The sample space of B = {H}

$$\therefore P(B) = \frac{1}{2}.$$

मान लिया $P(A \cap B)$ be the probability of getting the number 6 and a head.

$$\therefore P(A \cap B) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

(P.T.O.)

Using conditional probability

$P(A|B)$ = The probability that 6 come up given that head come up

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/12}{1/2} = \frac{1}{6}.$$

Q. A couple has 2 children. Find the probability that both are boys if it is known that

(i) One of the children is a boy, and
(ii) the elder child is a boy

Ans: एक couple के पास दो Children हैं।

The sample space

$S = \{(B, B), (B, G), (G, B), (G, G)\}$

मान लो कि $P(A)$ be the probability of both being boys.

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(i.) One of the children is a boy

माना की P(B) be the probability of one of them being a boy.

The sample space of B

$= \{(B,B), (B,G), (G,B)\}$

$\therefore P(B) = \frac{3}{4}$.

माना लिया कि $P(A \cap B)$ be the probability of one of them being a boy and both being boys.

$\therefore A \cap B = \{(B,B)\}$.

$\therefore P(A \cap B) = \frac{1}{4}$.

by using conditional probability :-

$P(A|B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/4}{3/4}$

$= \frac{1}{3}$

$\therefore$ Probability that both are boys if it is known as that one of the children is a boy

$= \frac{1}{3}$.

(ii) the elder child is a boy

मान लिया कि P(B) be the probability of the elder being a boy.

The sample space of B

$= \{(B, B), (B, G)\}$

$\therefore P(B) = \frac{1}{2}$

मान लिया कि $P(A \cap B)$ be the probability of the elder being a boy and both being boys.

$\therefore A \cap B = \{(B, B)\}$

$\therefore P(A \cap B) = \frac{1}{4}$

P(A|B) = probability of both being boy when the elder child is a boy.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/4}{1/2} = \frac{2}{4} = \frac{1}{2}$$

Q. In a class, 40% students study mathematics; 25% study biology and 15% study both maths and biology. One student is selected at random. Find the probability that :-

(i) he studies mathematics if it is known that he studies biology.
(ii) he studies biology if it is known that he studies mathematics.

Ans: माना कि P(A) be the probability of students studying mathematics.

$$\therefore P(A) = 40\% = \frac{40}{100} = 0.4$$

माना कि P(B) be the probability of students studying biology.

$$\therefore P(B) = 25\% = \frac{25}{100} = 0.25$$

माना कि $P(A \cap B)$ be the probability of students studying both mathematics and biology.

$$\therefore P(A \cap B) = 0.15$$

One student is selected at random.

$\because$ By conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

where P(A|B) is the probability of occurrence of event A given that B has already occurred.

(i.) he studies mathematics if it is known that he studies biology.

P(A|B) = probability that he studies mathematics given that he studies biology.

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.15}{0.25} = \frac{3}{5}$$

(ii.) he studies biology if it is known that he studies mathematics.

P(A|B) = probability that he studies biology given that he studies mathematics.

$$= P(A|B)$$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.15}{0.40} = \frac{3}{8}$$

Q. The probability that a student selected at random from a class will pass in Hindi is $\frac{4}{5}$ and the probability that he passes in Hindi and English is $\frac{1}{2}$. What is the probability that he will pass in English if it is known that he has passed in Hindi?

Ans: One student is selected at random.

माना P(A) be the probability of students passing in English.

माना P(B) be the probability of students passing in hindi.

$$\therefore P(B) = \frac{4}{5}$$

मान लिए कि $P(A \cap B)$ be the probability of students passing in both English & Hindi.

$$\therefore P(A \cap B) = \frac{1}{2}$$

by conditional probability

P(A|B) = The probability that he will pass in English given that he passes in Hindi

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/2}{4/5} = \frac{5}{8}$$

Q. The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a coat is 0.3 and the probability that he will buy a shirt given that he buys a coat is 0.4. Find the probability that he will buy both a shirt and a coat.

Ans: माना कि P(A) be the probability of a certain person buying a shirt.

$\therefore P(A) = 0.2$

माना P(B) be the probability of him buying a coat.

$\therefore P(B) = 0.3$

माना लिया कि $P(A \cap B)$ be the probability that he buys both a shirt and a coat.

by conditional probability

P(A|B) = the probability that he will buy a shirt given that he buys a coat.

$$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= 0.4$$

$\Rightarrow P(A \cap B) = P(A|B) \cdot P(B)$
$= 0.4 \times 0.3$
$= 0.12$

Q. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(i) Find the probability that he reads neither Hindi nor English news paper.

(ii) If he reads Hindi newspaper, what is the probability that he reads English newspaper?

(iii) If he reads English newspaper, what is the probability that he reads Hindi newspaper?

Ans: माना की P(A) be the probability of students reading hindi newspaper.

$\therefore P(A) = 60\% = \frac{60}{100} = 0.60$

माना की P(B) be the probability of them reading english newspaper.

$\therefore P(B) = 40\% = \frac{40}{100} = 0.40$

(P.T.O.)

माना की P(A∩B) be the probability of them reading both.

$\therefore P(A \cap B) = 0.20$.

मान लिए कि P(A∪B) be the probability of them reading either one of them.

$\therefore P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$
$= 0.60 + 0.40 - 0.20$
$= 1 - 0.20$
$= 0.80$

(i) The probability that he reads neither Hindi nor English newspaper

= 1 − probability that ~~either~~ of them reading either one of them

$= 1 - 0.8$
$= 0.2$
$= \frac{1}{5}$

(P.T.O.)

(ii.) The probability that she reads the english one given that she reads the hindi one

by using conditional probability

~~P(A|B)~~ P(B|A)

$$= \frac{P(A \cap B)}{P(A)} = \frac{0.20}{0.60} = \frac{1}{3}.$$

(iii.) The probability that she reads the Hindi one given that she reads the English one

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.20}{0.40} = \frac{1}{2}$$

Q. Two integers are selected at random from integers 1 through 11. If the sum is even, find the probability that both the numbers selected are odd.

Ans: Two integers are selected at random.

पहला choice के पास 11 options है 11 integers में से & second choice के पास 10 options है। बचे 10 integers में से

मान लिया कि, P(A) be the probability of choosing both numbers odd.

P(B) be the probability of choosing the numbers to yield an even number.

Sample Space of B

{(1,3) (1,5) (1,7) (1,9) (1,11) (3,5) (3,7)
(3,9) (3,11) (5,7) (5,9) (5,11) (7,9) (7,11)
(9,11), (2,4), (2,6), (2,8), (2,10), (4,6),
(4,8), (4,10), (6,8), (6,10), (8,10)}

$$\therefore P(B) = \frac{25}{11 \times 10} = \frac{25}{110}$$

Now let P(A∩B) be the probability of getting both odd numbers given that sum of digits is even.

$\therefore (A \cap B) =$

{(1,3), (1,5), (1,7), (1,9), (1,11), (3,5)
, (3,7), (3,9), (3,11), (5,7), (5,9)
, (5,11), (7,9), (7,11), (9,11)}

Properties of Conditional Probability :-

Let A and B be the events of a sample space 'S' of an experiment, then

(i) $P(S|A) = P(A|A) = 1$

(ii) If A and B are any two events of a sample space S and C is an event of S, such that $P(C) \neq 0$, then

$$P\{(A \cup B)/C\} = P(A|C) + P(B|C) - P\{(A \cap B)|C\}$$

In particular, if A and B are disjoint events, then

$$P\left(\frac{A \cup B}{C}\right) = P\left(\frac{A}{C}\right) + P\left(\frac{B}{C}\right)$$

(iii) $P(A'|B) = 1 - P(A|B)$, where A' is complement of A.

Note: If A and B are mutually exclusive events, then $P(A|B) = 0$

Multiplication theorem on Probability and Independent Events

मान लिया की A and B दो events हैं जो एक random experiment से associated हैं, तब

$$P(A \cap B) = P(A) \cdot P(B|A), \text{ where } P(A) \neq 0$$
$$= P(B) \cdot P(A|B), \text{ where } P(B) \neq 0$$

यहाँ, $A \cap B$ denote करती है simultaneous occurrence of the events A and B i.e. $A \cap B$ जो denote करता है कि ऐसा event जिसमें दोनों A और B occur हुआ हो।

उपरोक्त result को हम "Multiplication rule of Probability"

✯ Multiplication Theorem for more than Two Events

Let E, F, G and H be four events of sample space S. Then,

(i) $P(E \cap F \cap G) = P(E) \cdot P\left(\frac{F}{E}\right) \cdot P\left(\frac{G}{E \cap F}\right)$

(ii) $P(E \cap F \cap G \cap H)$

$$= P(E) \cdot P\left(\frac{F}{E}\right) \cdot P\left(\frac{G}{E \cap F}\right) \cdot P\left(\frac{H}{E \cap F \cap H}\right)$$

Independent Events

दो events A & B को independent कहेंगे अगर occurrence or non-occurrence of एक event का दूसरे event को affect नहीं करे।

दो events E and F are said to be independent, if

$$P\left(\frac{F}{E}\right) = P(F), \text{ provided } P(E) \neq 0$$

and

$$P\left(\frac{E}{F}\right) = P(E), \text{ provided } P(F) \neq 0$$

<u>दूसरे शब्दों में कहें तो :-</u>

अगर E & F दो events हैं, एक ही same random experiment के तो E & F को हम independent कहेंगे,

$$\text{if } P(E \cap F) = P(E) \cdot P(F).$$

(P.T.O.)

Important Results :-

1. दो events E and F को हम independent कहेंगे अगर वो independent नहीं है
   i.e. $P(E \cap F) \neq P(E) \cdot P(F)$

2. दो experiments को independent तब बोलेंगे जब हर एक pair of events E and F, जहाँ E को पहला experiment का event है & F जो है वह दूसरा experiment का even है तो probability of the simultaneous occurrence of the events E and F वो होगा product P(E) और P(F) का जिसको हम seperately calculate किए दो experiments के basis पर

   i.e. $P(E \cap F) = P(E) \cdot P(F)$

3. Three events A, B and C को mutually independent कहेंगे, अगर

$$P(A \cap B) = P(A) \cdot P(B)$$
$$P(A \cap C) = P(A) \cdot P(C)$$
$$P(B \cap C) = P(B) \cdot P(C)$$
and $$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

अगर ऊपर दिए गए चार में कोई भी सही नहीं हो तीन दिए गए events के लिए तो दिए गए events "independent नही" होंगे i.e. dependent होंगे।

Difference between Independent events & Mutually exclusive events

(i) "independent" term को हम define करते है probability of events के terms में लेकिन "mutually exclusive" को हम define करते है जब हम बात करते है कि जब events subset है sample space का।

(ii) Mutually exclusive events के पास कोई भी outcome common नही होता है लेकिन independent events के पास हो सकता है common outcome हो जाएँ

दूसरे शब्दों में :-
दो independent events जिनके पास non-zero probability of occurrence है वह कभी भी independent नहीं होती।

Properties of Independent Events :-

अगर A और B independent events है, तो

(i) A और B' भी independent events होंगी।

(ii) A' और B भी independent events होंगी।

(iii) A' और B' को भी independent events कहेंगी।

(P.T.O.)

Q. A bag contains 17 tickets, numbered from 1 to 17. A ticket is drawn, and then another ticket is drawn without replacing the first one. Find the probability that both the tickets may show even numbers.

Ans: एक बैग में 17 tickets हैं जिसमें 1 से 17 नंबर अंकित है और हर एक trial एक दूसरे से independent होते।

इसलिए sample space हुआ

$$S = \{1, 2, 3, \ldots\ldots, 17\}$$

हमको निकालना है the probability that both the tickets are drawn show even numbers.

तो ticket पर even no. आए इस event के लिए favourable outcome होंगे :-

$\{2, 4, 6, 8, 10, 12, 14, 16\}$

मान लिया, success : ticket drawn is even

अब, probability of success in the first trial

$$P_1(\text{success}) = \frac{8}{17}$$

probability of success in the second trial without replacement of the first draw is given by

$$P_2(\text{success}) = \frac{7}{16}$$

इसलिए,

probability that both the tickets show even numbers with each trial being independent is given by

$$P_1 \cap P_2 = P_1 \cdot P_2$$

$$= \frac{8}{17} \times \frac{7}{16_{2}} = \frac{7}{34}$$

Q. Two marbles are drawn successively from a box containing 3 black and 4 white marbles. Find the probability that both the marbles are black if the first marble is not replaced before the second draw.

Ans: दिया हुआ है कि :-
एक box में 3 black और 4 white marbles, हैं, हर एक trial एक दूसरे से independent है।

इसलिए sample space होगा

$$S = \{B_1, B_2, B_3, W_1, W_2, W_3, W_4\}$$

(P.T.O.)

तो हमको निकालना है कि probability that both the marbles drawn are black

माना कि, success : marble drawn is black

अत, the probability of success in the first trial is

$$P_1(\text{success}) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{\{B_1, B_2, B_3\}}{\{B_1, B_2, B_3, W_1, W_2, W_3, W_4\}} = \frac{3}{7}$$

Probability of success in the second trial without replacement of the first draw is given by

$$P_2(\text{success}) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{2}{6}$$ (∵ without replacement we are drawing again)

Hence,
the probability that both the marbles are drawn are black, with each trial being independent is given by

$$P(\text{success}_1 \cap \text{success}_2) = P_1(\text{success}) \times P_2(\text{success})$$

$$= \frac{3}{7} \times \frac{2}{6} = \frac{1}{7}.$$

Q. A card is drawn from a well-shuffled deck of 52 cards and without replacing this card, a second card is drawn. Find the probability that the first card is a club and the second card is a spade.

Ans: दिया हुआ है कि a well shuffled deck of 52 cards जिसमें हर draw एक दूसरे से independent होता है।

हमें निकालना है the probability that the first card is drawn is a club and the second card is a spade.

मान लिया कि

E: event that first card drawn is a club
H: event that second card drawn is a spade.

तो

$$P(E) = \frac{13}{52}$$ {∵ there is total of 13 cards in club suite

$$P(H) = \frac{13}{51}$$ {∵ there is total of 13 cards in spade suite & one card is already drawn

इसलिए,

the probability that the first card drawn is a club and the second card is a spade, with each trial being independent is given by

$$P(E \cap H) = P(E) \cdot P(H)$$

$$= \frac{13}{52} \times \frac{13}{51} = \frac{13}{204}$$

Q. There is a box containing 30 bulbs, of which 5 are defective. If two bulbs are chosen at random from the box in succession without replacing the first, what is the probability that both the bulbs are chosen are defective ?

Ans: दिया हुआ है कि एक box है जिसमें 30 bulbs है जिसमें 5 defective है, each trial एक दूसरे से independent है

तो हमको यह निकालना है कि the probability that both the bulbs are chosen are defective.

मान लिया कि

L: event that first ~~ball~~ bulb drawn is defective.

M: event that second bulb drawn is defective.

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तो

$P(L) = \frac{5}{30}$ { ∵ there are 5 defective balls and in total there are 30 bulbs.

$P(M) = \frac{4}{29}$ { ∵ there are remaining 4 defective bulbs and in total there are 29 bulbs

इसलिए,

the probability that both the bulbs are chosen are defective, with each trial being independent is given by

$$P(L \cap M) = P(L) \cdot P(M)$$

$$= \frac{5}{30} \times \frac{4}{29}$$

$$= \frac{2}{87}$$

Q. A bag contains 10 white and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that the first ball is white and the second is black?

Ans: दिया गया है कि एक bag में 10 white ball और 15 black balls हैं, each trial is independent of the other trial.

तो हमको निकालना है कि the probability that the first ball is drawn is white and the second ball drawn is black.

मान लिया,

E: event that first ball drawn is white.
M: event that second ball drawn is black.

$P(E) = \frac{10}{25}$ { ∵ there are total 25 balls out of which there are only 10 white balls.

$P(M) = \frac{15}{24}$ { ∵ there are total remaining 24 balls after first draw there are only 15 black balls.

Hence,

the probability that the first ball drawn is white & second ball drawn is black, with each trial being independent is given by.

$$P_1 \times P_2 = \frac{10}{25} \times \frac{15}{24} = \frac{1}{4}.$$

Q. An urn contains 5 white and 8 black balls. Two successive drawings of 3 balls at a time are made such that the balls drawn in the first draw are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and the second draw gives 3 black balls.

Ans: दिया गया है कि :-
एक कलश में 5 white balls और 8 black balls है; each trial is independent of the other trial.

तो हमको निकालना है, the probability that the first draws gives 3 white and the second draw gives 3 black balls.

तो मान लिए :-

E : event that the first draw gives 3 white balls

F : event that the second draw gives 3 black balls

$$P(E) = \frac{\text{selecting 3 balls out of 5 white balls}}{\text{selecting 3 balls out of total 13 balls}}$$

$$= \frac{{}^5C_3}{{}^{13}C_3} = \frac{\frac{5!}{2!\cdot 3!}}{\frac{13!}{10!\cdot 3!}}$$

$$= \frac{5}{143}$$

$$P(F) = \frac{\text{selecting 3 balls out of 8 black balls}}{\text{selecting 3 balls out of 10 remaining total balls}}$$

$$= \frac{{}^{8}C_{3}}{{}^{10}C_{3}} = \frac{\frac{8!}{5! \cdot 3!}}{\frac{10!}{7! \cdot 3!}} = \frac{8! \times 7! \times 3!}{10! \times 5! \times 3!}$$

$$= \frac{8! \times 7 \times 6 \times 5!}{10 \times 9 \times 8! \times 5!} = \frac{42}{90} = \frac{21}{45} = \frac{7}{15}$$

$$= \frac{7}{15}$$

इसलिए :

the probability that the first draw gives 3 white and the second draw gives 3 black balls, with each trial being independent is given by

$$P(E \cap F) = P(E) \cdot P(F)$$

$$= \frac{5}{143} \times \frac{7}{15_{3}}$$

$$= \frac{7}{429}$$

Q. let E_1 and E_2 be the events such that $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{3}{5}$.

find :-

(i) $P(E_1 \cup E_2)$, when E_1 and E_2 are mutually exclusive.

(ii) $P(E_1 \cap E_2)$, when E_1 and E_2 are independent.

Ans:

(i) $\because P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Since E_1 and E_2 are mutually exclusive events so, $E_1 \cap E_2 = \phi$

$$= \frac{1}{3} + \frac{3}{5} = \frac{5+9}{15} = \frac{14}{15}$$

$\therefore P(E_1 \cup E_2) = \frac{14}{15}$ when E_1 & E_2 are mutually exclusive.

(ii) $\because$ we know that E_1 and E_2 are independent.

So,

$$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$$

$$= \frac{1}{3} \times \frac{3}{5}$$

$$= \frac{1}{5}$$

Therefore, $P(E_1 \cap E_2) = \frac{1}{5}$ when E_1 and E_2 are independent.

Q. If E_1 and E_2 are the two events such that $P(E_1) = \frac{1}{4}$, $P(E_2) = \frac{1}{3}$ and $P(E_1 \cup E_2) = \frac{1}{2}$, show that E_1 and E_2 are independent events.

Ans: हमको बहुत अच्छे से पता है कि अगर दो event मान लिजिए A&B independent events है तो

$$P(A \cap B) = P(A) \cdot P(B) \text{ होगा}$$

पहले निकालो

$$\left[\because P(A \cap B) = P(A) + P(B) - P(A \cup B)\right]$$

$$P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1 \cup E_2)$$

$$= \frac{1}{4} + \frac{1}{3} - \frac{1}{2}$$

$$= \frac{3 + 4 - 6}{12} = \frac{1}{12}$$

2	4, 3, 2
2	2, 3, 1
3	1, 3, 1
	1, 1, 1

and,

$$P(E_1) \cdot P(E_2)$$

$$= \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

(P.T.O.)

Now,

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = \frac{1}{12}.$$

Hence therefore it is proved that E_1 & E_2 are independent events.

Q. If E_1 and E_2 are independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$, find

(i) $P(E_1 \cap E_2)$
(ii) $P(E_1 \cup E_2)$
(iii) $P(\overline{E_1} \cap \overline{E_2})$
(iv) $P(\overline{E_1} \cap E_2)$

Ans:

(i) क्योंकी E_1 & E_2 independent events हैं।
इसलिए,

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$
$$= 0.3 \times 0.4$$
$$= 0.12$$

(ii) $\because P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\Rightarrow 0.3 + 0.4 - 0.12$
$\Rightarrow 0.7 - 0.12$
$\Rightarrow 0.58$

(iii.) $P(\overline{E_1} \cap \overline{E_2})$

$\{ \because A' \cap B' = (A \cup B)'$; using de-morgan's law $\}$

$\Rightarrow P(\overline{E_1} \cap \overline{E_2})$

$\Rightarrow P(\overline{E_1 \cup E_2})$

$\Rightarrow 1 - P(E_1 \cup E_2)$

$\Rightarrow 1 - 0.58 = 0.42$

(iv.) $P(\overline{E_1} \cap E_2)$

$\Rightarrow P(\overline{E_1} \cap E_2)$

$\Rightarrow P(E_2) - P(E_1 \cap E_2)$

$\Rightarrow 0.4 - 0.12$

$\Rightarrow 0.28$

Q. Let A and B be the events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or not } B) = \frac{1}{4}$.

State wether A and B are

(i.) Mutually exclusive

(ii.) independent

(P.T.O.)

Ans: दिया गया है :—

$P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$

$P(\bar{A} \cup \bar{B}) = \frac{1}{4}$.

(i.) let consider that A & B be mutualy exclusive events

then $A \cap B = \phi$

i.e. $P(A \cup B) = P(A) + P(B) - \phi$

$P(A \cup B) = \frac{1}{2} + \frac{7}{12} -$ ——— (i)

but

$P(\bar{A} \cup \bar{B}) = P\overline{(A \cap B)} = \frac{1}{4}$

$=$ ~~ϕ~~ -

if we want

$P(A \cap B) = 1 - P\overline{(A \cap B)}$

$= 1 - \frac{1}{4} = \frac{3}{4}$.

from here

$P(A \cup B) = \frac{1}{2} + \frac{7}{12} - \frac{3}{4}$ ——— (ii)

Since, two distinctie values of P(A ∪ B) calculated in (i) & (ii) respectively contradicts.

So, ~~P(A~~ A & B are not mutually exclusive events.

(ii) if A & B would have to be independent events

then,

$$P(A \cap B) = P(A) \cdot P(B)$$

Now

$$P(A \cap B) = \frac{3}{4}$$

&

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$$

$$\because \quad P(A \cap B) \neq P(A) \cdot P(B).$$

So,

A and B are not independent events.

(P.T.O.)

Q. Kamal and Vimal appeared for an interview for two vacancies. The probability of Kamal's selection is 1/3, and that of Vimal's selection is 1/5. Find the probability that only one of them will be selected.

Ans: माना लिया :-

A: event that Kamal is selected.
B: event that Vimal is selected.

therefore,

$$P(A) = \frac{1}{3} \quad ; \quad P(B) = \frac{1}{5}.$$

हमें निकालना है probability कि सिर्फ एक आदमी select होगा।

The probability that only one of them will be selected

Now,

P(only one of them is selected)

= P(A and not B) or P(B and not A)

$= P(A \cap \bar{B}) + P(B \cap \bar{A})$

(P.T.O.)

∵ we know very well that the properties of independent events are :-
if A & B are independent event, then :-
$(A \cap B')$ & $(A' \cap B)$ are also independent events.

$$= P(A) \times P(\overline{B}) + P(B) \times P(\overline{A})$$

$$= P(A) \times [1 - P(B)] + P(B) \times [1 - P(A)]$$

$$= \frac{1}{3} \times \left(1 - \frac{1}{5}\right) + \frac{1}{5} \times \left(1 - \frac{1}{3}\right)$$

$$= \frac{1}{3} \times \frac{4}{5} + \frac{1}{5} \times \frac{2}{3}$$

$$= \frac{4}{15} + \frac{2}{15} = \frac{6}{15} = \frac{2}{5}$$

<u>Therefore,</u>

The probability that only one of them will be selected is $\frac{2}{5}$.

Q. Arun and Ved appeared for an interview for two vacancies. The probability of Arun's selection is 1/4, and that of Ved's rejection is 2/3. Find the probability that at least one of them will be selected.

Ans: माना किया :-

E : event that arun get selected
Z : event that ved got selected

therefore,

$$P(E) = \frac{1}{4}, \quad P(Z) = \frac{2}{3}$$

∵ E & Z are independent events so E' & Z ; E & Z' are also independent events.

$P(\bar{E}) = 1 - P(E)$	$P(\bar{Z}) = 1 - P(Z)$
$= 1 - \frac{1}{4}$	$= 1 - \frac{2}{3}$
$= \frac{4-1}{4} = \frac{3}{4}$	$= \frac{1}{3}$

∴ Probability that at least one of them will be selected

$$= P(E \cap Z') + P(E' \cap Z) + P(E \cap Z)$$

= probability that arun selected & ved not selected (or)
probability that ved selected & arun not selected (or)

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probability that both arun & ved get selected.

$= P(E\cap Z') + P(E'\cap Z) + P(E\cap Z)$

$= \{P(E) - P(E\cap Z)\} + \{P(Z) - P(E\cap Z)\}$

$+ P(E\cap Z)$

$= \left\{\frac{1}{4} - P(E)\cdot P(Z)\right\} + \{P(Z) - P(E)\cdot P(Z)\}$

$+ P(E)\cdot P(Z)$

$= \left\{\frac{1}{4} - \frac{1}{4}\times\frac{2}{3}\right\} + \left\{\frac{1}{3} - \frac{1}{4}\times\frac{1}{3}\right\} + \left\{\frac{1}{4_2}\times\frac{2}{3}\right\}$

$= \left\{\frac{1}{4} - \frac{2}{12_6}\right\} + \left\{\frac{1}{3} - \frac{1}{12}\right\} + \frac{1}{6}$

$= \left\{\frac{1}{4} - \frac{1}{6}\right\} + \left\{\frac{1}{3} - \frac{1}{12}\right\} + \frac{1}{6}$

$= \left\{\frac{3-2}{12}\right\} + \left\{\frac{4-1}{12}\right\} + \frac{1}{6}$

$= \frac{1}{12} + \frac{3}{12} + \frac{1}{6} = \frac{4}{12} = \frac{1}{3} + \frac{1}{6}$

$= \frac{6+3}{18} = \frac{9}{18} = \frac{1}{2}$

Q. A and B appear for an interview for two vacancies in the same post. The probability of A's selection is 1/6 and that of B's selection is 1/4. Find the probability that :-

(i.) both of them are selected
(ii.) only one of them is selected
(iii.) none is selected
(iv.) at least one of them is selected.

Ans: माना लिया कि :-

E: event that A get selected.
J: event that B get selected.

दिया गया है कि :-

$$P(A) = \frac{1}{6} \quad ; \quad P(B) = \frac{1}{4}.$$

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{6} = \frac{5}{6}$$

$$P(\bar{A}) = \frac{5}{6}$$

$$P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$$

(P.T.O.)

(i) both of them are selected

$\because$ A & B are independent events.

So,

$P(A \cap B)$ = probability of getting both of them selected.

$$= P(A) \cdot P(B)$$

$$= \frac{1}{6} \times \frac{1}{1} = \frac{1}{24}.$$

(ii) Only one of them is selected.

$$P(A' \cap B) + P(A \cap B')$$

$$= \{P(B) - P(A \cap B)\} + \{P(A) - P(A \cap B)\}$$

$$= \left(\frac{1}{4} - \frac{1}{24}\right) + \left(\frac{1}{6} - \frac{1}{24}\right).$$

$$= \left(\frac{6-1}{24}\right) + \left(\frac{4-1}{24}\right)$$

$$= \frac{5}{24} + \frac{3}{24} = \frac{8}{24} = \frac{1}{3}.$$

Therefore, the probability that only one of them is selected

$$= \frac{1}{3}$$

(iii) none is selected

Probability of none get selected

$= P(\bar{A} \cap \bar{B})$

$= P(\bar{A}) \cdot P(\bar{B})$

$= \frac{5}{6} \times \frac{3}{4}$

$= \frac{15}{24} = \frac{3}{8} = \frac{3}{8}$.

[∵ A & B are independent events, so, A' & B' are also independent events.]

Therefore, the probability that atleast one of them is selected is $\frac{3}{8}$.

(iv) at least one of them is selected.

P(at least one of them is selected)

= P(selecting only A) +
P(selecting only B) +
P(selecting both A & B).

= P(A and not B) + P(B and not A) +
P(A and B)

$$= P(A)\cdot P(\overline{B}) + P(B)\cdot P(\overline{A}) + P(A)\cdot P(B)$$

$$= \left(\frac{1}{6}\times\frac{3}{4}\right) + \left(\frac{1}{4}\times\frac{5}{6}\right) + \frac{1}{6}\times\frac{1}{4}.$$

$$= \frac{1}{8} + \frac{5}{24} + \frac{1}{24}$$

$$= \frac{3+5+1}{24} = \frac{9}{24} = \frac{3}{8}$$

Q. Given the probability that A can solve a problem is 2/3, and the probability that B can solve the same problem is 3/5, find the probability that :—

(i) at least one of A and B will solve the problem

(ii) none of the two will solve the problem.

Ans:

Let,

E: event that A can solve the problem.

M: event that B can solve the problem.

$$P(E) = \frac{2}{3}$$

$$P(\overline{E}) = 1 - \frac{2}{3} = \frac{1}{3}$$

$$P(M) = \frac{3}{5}$$

$$P(\overline{M}) = 1 - \frac{3}{5} = \frac{2}{5}$$

Clearly, A & B are independent events
so, A' & B' are also independent events.

(ii) at least one of A and B will solve the problem

Sol,

P(at least one of A and B will solve the problem.)

$= 1 - P$(both are unable to solve the problem.)

$= 1 - P(A' \cap B')$

$= 1 - P(\overline{A}) \times P(\overline{B})$

$= 1 - P(\overline{E}) \times P(\overline{M})$

$= 1 - \frac{1}{3} \times \frac{2}{5}$

$= 1 - \frac{2}{15} = \frac{13}{15}$

Therefore, atleast one of A and B will solve the problem is $\frac{13}{15}$.

(ii) none of the two will solve the problem.

अतः,

P(none of the two will solve the problem)

$= P(\bar{A} \cap \bar{B})$

$= P(\bar{A}) \cdot P(\bar{B})$

$= P(\bar{E}) \cdot P(\bar{M})$

$= \frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$.

Q. A problem is given to three students whose chances of solving it are 1/4, 1/5 & 1/6, respectively. Find the probability that the problem is solved.

Ans: दिया हुआ है कि :-

let A: be the event that problem solved by first student.

B: be the event that problem solved by second student.

C: be the event that problem solved by third student.

(P.T.O.)

Now,

$P(A) = \frac{1}{4}$, $P(B) = \frac{1}{5}$ & $P(C) = \frac{1}{6}$

$P(\overline{A}) = 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$

$P(\overline{B}) = 1 - \frac{1}{5} = \frac{5-1}{5} = \frac{4}{5}$

$P(\overline{C}) = 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$.

∴ The probability that the problem is solve

P(the problem is solved)

= 1 − P(the problem is not solved)

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 - P(\overline{A}) \times P(\overline{B}) \times P(\overline{C})$

$= 1 - \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}$

$= 1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$.

∴ The probability that the problem is solved

$= \frac{1}{2}$.

Q. The probabilities of A, B, C solving a problem are 1/3, 1/4 and 1/6, respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

Ans: माना कि ~~A, B and C be three students~~

E : event that A solves the problem.
F : event that B solves the problem.
Z : event that C solves the problem.

$P(E) = \frac{1}{3}$, $P(F) = \frac{1}{4}$, $P(Z) = \frac{1}{6}$

$P(\overline{E}) = 1 - P(E)$	$P(\overline{F}) = 1 - P(F)$	$P(\overline{Z}) = 1 - P(Z)$
$= 1 - \frac{1}{3}$	$= 1 - \frac{1}{4}$	$= 1 - \frac{1}{6}$
$= \frac{3-1}{3}$	$= \frac{4-1}{4}$	$= \frac{6-1}{6}$
$= \frac{2}{3}$	$= \frac{3}{4}$	$= \frac{5}{6}$

Now,

P(exactly one of them will solve it)

= P(A and not B and not C) +
P(B and not A and not C) +
P(C and not A and not B).

(P.T.O.)

$$= P(E \cap \bar{F} \cap \bar{Z}) + P(\bar{E} \cap F \cap \bar{Z}) + P(\bar{E} \cap \bar{F} \cap Z)$$

{ ∵ E, F, Z are independent events then E', F', Z' are also independent events }

$$= P(E) \times P(\bar{F}) \times P(\bar{Z}) + P(\bar{E}) \times P(F) \times P(\bar{Z}) + P(\bar{E}) \times P(\bar{F}) \times P(Z)$$

$$= \left(\frac{1}{3} \times \frac{3}{4} \times \frac{5}{6}\right) + \left(\frac{2}{3} \times \frac{3}{4} \times \frac{5}{6}\right) + \left(\frac{2}{3} \times \frac{3}{4} \times \frac{1}{6}\right)$$

2	24, 12, 12
2	12, 6, 6
2	6, 3, 3
3	2, 1, 1
	1, 1, 1

$$= \frac{5}{24} + \frac{5}{12} + \frac{1}{12}$$

$$= \frac{5 + 10 + 2}{24} = \frac{17}{24}$$

$$= \left(\frac{1}{3} \times \frac{3}{4} \times \frac{5}{6}\right) + \left(\frac{1}{4} \times \frac{2}{3} \times \frac{5}{6}\right) + \left(\frac{1}{6} \times \frac{3}{4} \times \frac{2}{3}\right)$$

$= \frac{15}{72} + \frac{10}{72} + \frac{6}{72} = \frac{31}{72}.$

Therefore,
The probability that exactly one of them will solve the problem is $\frac{31}{72}$.

Q. A can hit a target 4 times in 5 shots, B can hit 3 times in 4 shots, and C can hit 2 times in 3 shots. Calculate the probability that:-

(i.) A, B and C all hit the target.
(ii.) B and C hit and A does not hit the target.

Ans: (i.) Let,

K: event that A hit a target once.
R: event that B hit a target once.
M: event that C hit a target once.

$$P(K) = \frac{\text{no. of times A hit the shot}}{\text{total no. of shots}} = \frac{4}{5}$$

$$P(R) = \frac{\text{no. of times B hit the shot}}{\text{total no. of shots}} = \frac{3}{4}$$

$$P(M) = \frac{\text{no. of times C hit the shot}}{\text{total no. of shots}} = \frac{2}{3}$$

Now,

$$P(\overline{K}) = 1 - P(K)$$
$$= 1 - \frac{4}{5} = \frac{5-4}{5} = \frac{1}{5}$$

$$P(\overline{R}) = 1 - P(R)$$
$$= 1 - \frac{3}{4} = \frac{4-3}{4} = \frac{1}{4}$$

$$P(\overline{M}) = 1 - P(M)$$
$$= 1 - \frac{2}{3} = \frac{3-2}{3} = \frac{1}{3}$$

Clearly, ~~A, B & C~~ K, R & M are independent events so also K', R & M are independent events.

∴ Probability that A, B & C all hit the target

$$= P(K \cap R \cap M)$$
$$= P(K) \cdot P(R) \cdot P(M)$$
$$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$$

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Hence, The probability that A, B and C all hit the target is $\frac{2}{5}$.

(ii.)

Probability that B and C hit and A does not hit the target.

P(B and C hit and not A)

$= P(\overline{K} \cap R \cap M)$

$= P(\overline{K}) \times P(R) \times P(M)$

~~$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$~~

$= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{10}$.

Hence, the probability that B and C hit and A does not hit the target is $\frac{1}{10}$.

Q. Neelam has offered physics, chemistry & mathematics in class XII. She estimates that her probabilities of receiving a grade A in these courses are 0.2, 0.3 and 0.9 respectively. Find the probabilities that Neelam recieves

(i.) all A grades
(ii.) no A grade
(iii.) exactly 2 A grades.

Ans: मान लिया कि:-

E : event that neelam getting A grade in physics.
F : event that neelam getting A grade in chemistry.
G : event that neelam getting A grade in mathematics.

$P(E) = 0.2$, $P(F) = 0.3$, $P(G) = 0.9$

$P(\overline{E}) = 1 - P(E)$
$= 1 - 0.2$
$= 0.8$

$P(\overline{F}) = 1 - P(F)$
$= 1 - 0.3$
$= 0.7$

$P(\overline{G}) = 1 - P(G)$
$= 1 - 0.9 = 0.1.$

(i) The probability that neelam gets all A grades

Here,

P(getting all A grades)

$= P(E \cap F \cap G)$

$= P(E) \cdot P(F) \cdot P(G)$

$= 0.2 \times 0.3 \times 0.9$

$= 0.054$

∴ The probability that neelam gets all A grades is 0.054.

(ii) The probability that neelam gets no A grades

P(getting no A grade)

$= P(\overline{E} \cap \overline{F} \cap \overline{G})$

$= P(\overline{E}) \times P(\overline{F}) \times P(\overline{G})$

$= 0.8 \times 0.7 \times 0.1$

$= 0.056$

∴ The probability that neelam gets no A grades is 0.056.

(iii) The probability that neelam gets exactly 2 A grades

P(getting exactly 2 A grades)

= P(A and B and not C) + P(B and C and not A) + P(C and A and not B)

$= P(E \cap F \cap \overline{G}) + P(F \cap G \cap \overline{E}) + P(G \cap E \cap \overline{F})$

$= [0.2 \times 0.3 \times 0.1] + [0.8 \times 0.9 \times 0.3]$
$+ [0.9 \times 0.2 \times 0.7]$

$= [0.006 + 0.216 + 0.126]$
$= 0.348$

∴ The probability that neelam gets exactly 2A grades is

0.348.

Q. An article manufactured by a company consists of two parts X and Y. In the process of manufacture of part X, 8 out of 100 parts may be defective. Similarly, 5 out of 100 parts of Y may be defective. Calculate the probability that the assembled product will not be defective.

Ans: मान लिया

R: is the event that a defective part manufactured in X.
Q: is the event that a defective part manufactured in Y.

Now,

$P(R) = \frac{8}{100}$ | $P(Q) = \frac{5}{100}$

$P(\overline{R}) = 1 - P(R)$

$= 1 - \frac{8}{100}$

$= \frac{100-8}{100}$

$= \frac{92}{100}$

$P(\overline{Q}) = 1 - P(Q)$

$= 1 - \frac{5}{100}$

$= \frac{100-5}{100}$

$= \frac{95}{100}$

∴ Probability that the assembled product will not be defective

= 1 − P(product assembled to be defective)

= 1 − [P(X and not Y) + P(Y and not X) + P(both X and Y)]

$= 1 - [P(R \cap \overline{Q}) + P(Q \cap \overline{R}) + P(R \cap Q)]$

$= 1 - [P(R) \cdot P(\overline{Q}) + P(Q) \cdot P(\overline{R}) + P(R) \cdot P(Q)]$

$= 1 - [\frac{8}{100} \times \frac{95}{100} + \frac{5}{100} \times \frac{92}{100} + \frac{8}{100} \times \frac{5}{100}]$

$= 1 - [\frac{760}{10000} + \frac{460}{10000} + \frac{40}{10000}] = \frac{437}{500}$

Therefore,
The probability that the assembled product will not be defective is

$$\frac{437}{500}$$

Q. A town has two fire-extinguishing engines, functioning independently. The probability of availability of each engine when needed is 0.95. What is the probability that

(i.) neither of them is available when needed?
(ii.) an engine is available when needed?

Ans: मान लिया कि,

E : event that first F.E.E is available
G : event that second F.E.E is available
$P(E) = 0.95$ & $P(G) = 0.95$

(i.) The probability that neither of them is available when needed.

$P(\text{not } E \text{ and not } G)$

$= P(\bar{E} \cap \bar{G})$

$= P(\bar{E}) \times P(\bar{G})$

$= [1 - P(E)] \times [1 - P(G)]$

{∵ E & G are independent events then E' & G' are independent events}

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$= [1 - 0.95] \times [1 - 0.95]$

$= 0.05 \times 0.05$

$= 0.0025$

∴ The probability that neither of them is available when needed

$= 0.0025$.

(ii.) The probability that an engine is available when needed:

Hence,

P(E and not G) (or) P (not E and G)

$= P(E \cap \bar{G}) + P(\bar{E} \cap G)$

$= P(E) \cdot P(\bar{G}) + P(\bar{E}) \cdot P(G)$

$= 0.95 \times 0.05 + 0.05 \times 0.95$

$= 0.0475 + 0.0475$

$= 0.0950$

∴ The probability that an engine is available when needed is

$= 0.0950$.

Q. A machine operates only when all of its three components function. The probabilities of the failures of the first, second and third components are 0·14, 0·10 and 0·05, respectively. What is the probability that the machine will fail?

Ans: मान लिया कि :-

E: event that the first component fails.
B: event that the second component fails.
Z: event that the third component fails.

$P(E) = 0{\cdot}14$; $P(B) = 0{\cdot}10$; $P(Z) = 0{\cdot}05$

$P(\overline{E}) = 1 - 0{\cdot}14 = 0{\cdot}86$

$P(\overline{B}) = 1 - 0{\cdot}10 = 0{\cdot}90$

$P(\overline{Z}) = 1 - P(Z) = 1 - 0{\cdot}05 = 0{\cdot}95$

Hence,

The probability that the machine will fail

~~= P(E ∩ B ∩ Z)~~

= 1 − P(the machine will function)

$= 1 - P(\text{all three components are working})$

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 - [P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})]$

$= 1 - [P(\bar{E}) \times P(\bar{B}) \times P(\bar{Z})]$

$= 1 - [0.86 \times 0.90 \times 0.95]$

$= 1 - 0.7353$

$= 0.2647$

Therefore,
The probability that the machine will fail is 0.2647.

Q. An anti-air-craft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that at least one shot hits the plane?

(P.T.O.)

Ans:

B: event of hitting the plane at the first shot.
Q: event of hitting the plane at the second shot.
R: event of hitting the plane at the third shot.
S: event of hitting the plane at the fourth shot.

$P(B) = 0.4$, $P(Q) = 0.3$, $P(R) = 0.2$ & $P(S) = 0.1$

$P(\bar{B}) = 1 - P(B)$
$= 1 - 0.4$
$= 0.6$

$P(\bar{Q}) = 1 - P(Q)$
$= 1 - 0.3$
$= 0.7$

$P(\bar{R}) = 1 - P(R)$
$= 1 - 0.2$
$= 0.8$

$P(\bar{S}) = 1 - 0.1$
$= 0.9$

The probability that atleast one shot hits the plane.

$\Rightarrow$ P(atleast one shot hits the plane)
= 1 − P(none of the shot hit the plane)

$= 1 - P(\bar{B} \cap \bar{Q} \cap \bar{R} \cap \bar{S})$

$= 1 - P(\bar{B}) \cdot P(\bar{Q}) \cdot P(\bar{R}) \cdot P(\bar{S})$

$= 1 - 0.6 \times 0.7 \times 0.8 \times 0.9$
$= 1 - 0.3024$
$= 0.6976$

Therefore,
The probability that atleast one shot hits the plane is
$= 0.6976$

Theorem of Total Probability and Baye's Theorem

एक set of events $E_1, E_2, \ldots, E_n$ is said to represent a partition of the sample space S, अगर वह following conditions को satisfy करेगा :—

(Partition of sample space)

(i) $E_i \cap E_j = \phi$, $i \neq j$; $j, i = 1, 2, \ldots, n$

(ii) $E_1 \cup E_2 \cup \ldots \cup E_n = S$

(iii) $P(E_i) > 0$, $\forall i = 1, 2, \ldots, n$

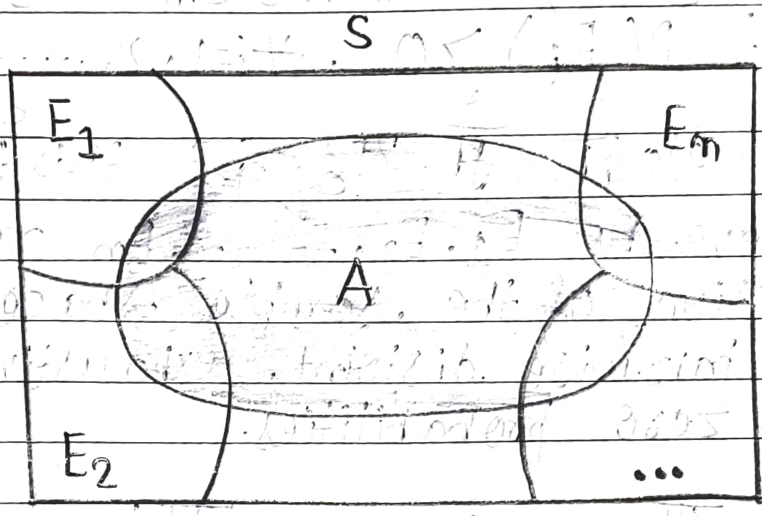
दूसरे शब्दों में यह कह सकते हैं कि :— the events $E_1, E_2, \ldots, E_n$ represent a partition of the sample space S, if they are pairwise disjoint, exhaustive and have non-zero probabilities.

Note : एक sample space का partition unique नहीं होता है, एक sample space का बहुत सारा partitions हो सकता है।

(P.T.O.)

(Theorem of Total Probability)

मान लिया कि $\{E_1, E_2, E_3, \ldots, E_n\}$ partions हैं sample space S के और मान लिया कि events $E_1, E_2, E_3, \ldots, E_n$ के पास non-zero probability of occurrence है और मान लिया कि A कोई event है जो S से associated है, तब



$$P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + \ldots + P(E_n) \cdot P(A|E_n)$$

$$= \sum_{j=1}^{n} P(E_j) \cdot P(A|E_j)$$

Bayes' Theorem

अगर $E_1, E_2, \ldots, E_n$ 'n' non-empty events है जो एक partition बनाता है sample space S का i.e. $E_1, E_2, \ldots, E_n$ are pairwise disjoint.

$E_1 \cup E_2 \cup \ldots \cup E_n = S$ and

$P(E_i) > 0, \forall \; i = 1, 2, 3, \ldots, n.$

Also, let A be any event of non-zero probability, तब

$$P(E_i | A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A|E_i)}, \text{ for any } i = 1, 2, 3, 4, \ldots, n$$

(Or),

मान लो कि $E_1, E_2, E_3, \ldots, E_n$ be 'n' mutually exclusive and exhaustive events जो एक random experiment का हिस्सा है। अगर A एक event है जो E_i's के साथ-साथ occur होगा. तब,

$$P(E_i | A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A|E_i)}$$

(P.T.O.)

- events $E_1, E_2, \ldots, E_n$ को hypothesis कहेंगे।
- probability $P(E_i)$ को priori probability of the hypothesis E_i कहेंगे।
- Conditional probability $P(E_i|A)$ को posteriori probability of the hypothesis E_i कहेंगे।

<u>Note</u>: Baye's Theorem is also called the formulae for the probability of causes.

Method to solve problem based on Baye's Theorem

baye's theorem पर based problems को solve करने के लिए हम निम्नलिखित steps follow करेंगे :-

I. सभी events को identify करें और उसको naming करें E_1, E_2, E_3 and so on, जो कि एक partition बनाएगा sample space S को।

II. मान लिया की दूसरा event जिसका non-zero probability है वह 'A' है।

III. सारे events $E_1, E_2, E_3 \ldots\ldots$ का probability निकालेंगे।

IV. A का सारे events $E_1, E_2, E_3, \ldots\ldots$ के साथ Conditional probability निकालेंगे।

V. Bayes theorem का use करके हम required probability निकालेंगे i.e. $P(E_i | A)$.

Q. In a bulb factory, three machines A, B, C, manufacture 60%, 25%, and 15% of the total production respectively. of their respective outputs 1%, 2% and 1% are defective. A bulb is drawn at random from the total product, and it is found to be defective. Find the probability that it was manufactured by machine C.

Ans: Step 1: Let,

E_1: event that bulb is manufactured from ~~factory~~ machine A.

E_2: event that bulb is manufactured from machine B.

E_3: event that bulb is manufactured from machine C.

Step 2: let,

A: event that the bulb drawn ~~drew~~ is defective.

(P.T.O.)

Step 3:

$P(E_1) = 60\% = \frac{60}{100} = \frac{3}{5}$

$P(E_2) = 25\% = \frac{25}{100} = \frac{1}{4}$

$P(E_3) = 15\% = \frac{15}{100} = \frac{3}{20}$

Step 4:

$P(A|E_1)$ = probability of defective bulb is made by machine A

$= \frac{1}{100}$

$= \frac{\text{favourable outcomes that defective bulb is made by machine A}}{\text{total bulbs produced}}$

$= \frac{1\%}{100\%} = \frac{1/100}{100/100}$

$= \frac{1}{100}.$

$P(A|E_2)$ = probability of defective bulb is made by machine B.

$$= \frac{\text{favourable outcomes that defective bulb is made by machine B.}}{\text{total bulbs produced.}}$$

$$= \frac{2\%}{100\%} = \frac{2/100}{100/100} = \frac{2}{100}$$

$P(A|E_3)$ = probability of defective bulb is made by machine C.

$$= \frac{\text{favourable outcomes that defective bulb is made by machine C.}}{\text{total bulbs produced.}}$$

$$= \frac{1\%}{100\%} = \frac{1/100}{100/100} = \frac{1}{100}$$

Step 5:

Now, using the bayes's theorem, we get

$P(E_3|A)$ = probability that the selected defective bulb is manufactured by C

$$= \frac{P(E_3) \cdot P(A|E_3)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)}$$

$$= \frac{\frac{3}{20} \times \frac{1}{100}}{\frac{3}{5} \times \frac{1}{100} + \frac{1}{4} \times \frac{2}{100} + \frac{3}{20} \times \frac{1}{100}}$$

$$= \frac{\frac{3}{2000}}{\frac{3}{500} + \frac{\cancel{2}}{\cancel{400}\ 200} + \frac{3}{2000}}$$

$$= \frac{\frac{3}{2000}}{\frac{12 + 10 + 3}{2000}}$$

$$= \frac{3}{25}$$

Conclusion:
Therefore, the probability of selected defective bulb is from machine C

$$= \frac{3}{25}.$$

Q. A company manufactures scooters at two plants, A and B. plant A produces 80% and plant B produces 20% of the total product. 85% of scooters produced at plant A and 65% of the scooters produced at plant B are of standard quality. A scooter produced by the company is

selected at random, and it is found to be of standard quality. What is the probability that it was manufactured at plant A ?

Ans:

Step 1: Let,

E_1: event that a scooter is manufactured at plant A.

E_2: event that a scooter is manufactured at plant B.

Step 2: Let,

A: event that scooter is of standard quality.

Step 3:

$P(E_1) = \frac{80}{100} = \frac{4}{5}$

$P(E_2) = \frac{20}{100} = \frac{1}{5}$

Step 4:

$P(A|E_1)$ = probability that the scooter is of standard quality made by plant A.

= favourable outcomes that scooter is of standard quality made by plant A / total scooters produced.

$$= \frac{\frac{85}{100}}{\frac{100}{100}} = \frac{85}{100}$$

$P(A|E_2)$ = probability that the scooter is of standard quality made by plant B.

$$= \frac{\frac{65}{100}}{\frac{100}{100}} = \frac{65}{100}$$

Step 5:

Now, using bayes's theorem, we get

$P(E_1|A)$ = probability that selected standard scooter is from plant A.

$$= \frac{P(E_1)\cdot \cancel{P(E_1|A)}\ (A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{4}{5}\times\frac{85}{100}}{\frac{4}{5}\times\frac{85}{100} + \frac{1}{5}\times\frac{65}{100}}$$

(85 → 17, 65 → 13)

$$= \frac{\frac{17\times4}{100}}{\frac{17\times4+13}{100}} = \frac{17\times4}{17\times4+13}$$

$$= \frac{68}{68+13} = \frac{68}{81}$$

Q. In a certain college, 4% of boys and 1% of girls are taller than 1.75 meters. Furthermore, 60% of the students are girls. If a student is selected at random and is taller than 1.75 meters, what is the probability that the selected student is a girl?

Ans:

~~Step 1: Let,~~

~~E_1: event that a boy is taller than 1.75 mts.~~
~~E_2: event that a girl is taller than 1.75 mts.~~

~~Step 2:~~

Step 1:
Let,

E_1: event that a student is a boy.
E_2: event that a student is a girl.

Step 2:
Let,

A: event that student is taller than 1.75 meters.

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Step 3:

$P(E_1) = 40\% = \frac{40}{100}$

$P(E_2) = \text{~~40\%~~} 60\% = \frac{60}{100}$

Step 4:

$P(A|E_1)$ = Probability that the student is taller than 1.75 meters is a boy

= favourable outcomes to the student boy is taller than 1.75 mts. / total students.

$= \frac{4\%}{100\%}$

$= \frac{4/100}{100/100} = \frac{4}{100}$

$P(A|E_2)$ = Probability that the student is taller than 1.75 meters is a girl.

= favourable outcomes to the student girl is taller than 1.75 mts. / total students

$$= \frac{1\%}{100\%} = \frac{1/100}{100/100} = \frac{1}{100}$$

Step 5:

Now, using the baye's theorem

$P(E_2|A)$ = probability that selected student is a girl and is taller than 1.75 meters.

$$= \frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\left(\frac{60}{100}\right)\times\left(\frac{1}{100}\right)}{\left(\frac{40}{100}\right)\times\left(\frac{4}{100}\right) + \left(\frac{60}{100}\right)\times\left(\frac{1}{100}\right)}$$

$$= \frac{\frac{3}{500}}{\frac{2}{125} + \frac{3}{500}}$$

5	500, 125
5	100, 25
5	20, 5
2	4, 1
2	2, 1
	1, 1

$$= \frac{\frac{3}{500}}{\frac{8+3}{500}} = \frac{3}{11}$$

$125 \times 4 = 500$

classmate

Conclusion: Therefore, the probability of selected taller student is a girl is $\frac{3}{11}$.

Q. In a class, 5% of the boys and 10% of the girls have an IQ of more than 150. In this class, 60% of the students are boys. If a student is selected at random and found to have IQ more than 150, find the probability that the student is a boy.

Ans: Step 1:

Let,

E_1: event that a student is a boy.

E_2: event that a student is a girl.

Step 2:

Let,

A: event that student has IQ more than 150.

Step 3:

$P(E_1) = ~~5\%~~ 60\%$ $\qquad$ $P(E_2) = 40\%$

$= \frac{~~5~~ 60}{100}$ $\qquad$ $= \frac{40}{100}$

Step 4:

$P(A|E_1)$ = Probability that the student has IQ greater than 150 is a boy.

$$= \frac{\text{favourable outcomes to the student boy has more than 150 IQ.}}{\text{total no. of students.}}$$

$$= \frac{5\%}{100\%} = \frac{5/100}{100/100} = \frac{5}{100}$$

$P(A|E_2)$ = Probability that the student has IQ greater than 150 is a girl.

$$= \frac{\text{favourable outcomes to the student girl having IQ more than 150.}}{\text{total no. of students}}$$

$$= \frac{10\%}{100\%} = \frac{10/100}{100/100} = \frac{10}{100} = \frac{1}{100}$$

(P.T.O.)

Step 5:

Now, using the bayers theorem

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{60}{100}\times\frac{5}{100}}{\frac{60}{100}\times\frac{5}{100}+\frac{40}{100}\times\frac{10}{100}}$$

$$= \frac{\frac{3}{100}}{\frac{3}{100}+\frac{10}{250}}$$

$$= \frac{\frac{3}{100}}{\frac{15+2\times10}{500}}$$

$$= \frac{3\times500}{17\times100}$$

$$= \frac{\frac{3}{100}}{\frac{15+20}{500}}$$

$$= \frac{3 \times \cancel{500}}{\cancel{100} \times \cancel{35}\ 7} = \frac{3}{7}.$$

Conclusion: Therefore, the probability that selected student having IQ greater than 150 is a boy is $\frac{3}{7}$.

Q. Suppose 5% of men and 0.25% of women have grey hair. A grey-haired person is selected at random. What is the probability of this person being male? Assume that there is an equal no. of males and females.

Ans: Step 1:

मान लिया कि

E_1: event that person is a men

E_2: event that person is a woman.

Step 2:

Let,

A: event that the person have grey hair.

Step 3:

$$P(E_1) = \frac{50\%}{100\%} = \frac{50/\cancel{100}}{100/\cancel{100}} = \frac{50}{100}$$

$$P(E_2) = \frac{50\%}{100\%} = \frac{50/\cancel{100}}{100/\cancel{100}} = \frac{50}{100}$$

Step 4:

$P(A|E_1)$ = probability that person have grey hair is a man.

$$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{5\%}{100\%} = \frac{\frac{5}{100}}{100/100} = \frac{5}{100}$$

$P(A|E_2)$ = probability that person have grey hair is a woman.

$$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{\frac{0.25}{100}}{\frac{100}{100}} = \frac{0.25}{100}$$

Step 5:

Now, using the baye's theorem

$$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)}$$

$$= \frac{\frac{50}{100} \times \frac{5}{100}}{\frac{50}{100} \times \frac{5}{100} + \frac{50}{100} \times \frac{0.25}{100}}$$

$$= \frac{\frac{1}{2} \times \frac{5}{100}}{\left(\frac{1}{2}\right) \times \left(\frac{5}{100}\right) + \left(\frac{1}{2}\right) \times \left(\frac{0.25}{100}\right)}$$

$$= \frac{5}{5.25} = \frac{20}{21}$$

Conclusion: Therefore, the probability of a randomly selected grey person to be male is $\frac{20}{21}$.

Q. Two groups are ~~connecting~~ competing for the positions on the board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4, respectively. Further, if the first group wins, the probability of introducing a new product is 0.7, and when the second groups win, the corresponding probability is 0.3. Find the probability that the new product introduced was by the second group.

(P.T.O.)

Ans: let,

E_1: be the event that first group will win.
E_2: be the event that second group will win

A: be the event that new product is introduced.

$P(E_1) = 0.6$ $P(E_2) = 0.4$

$P(A|E_1)$ = probability that new product is introduced when first group will wins.

$$= \frac{0.7}{1}$$

$P(A|E_2)$ = probability that new product is introduced when second group will wins.

$$= \frac{0.3}{1}$$

by using baye's theorem, we get

$$P(E_2|A) = \frac{P(E_2) \cdot P(A|E_2)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

$$= \frac{(0.4 \times 0.3)}{(0.6 \times 0.7) + (0.4 \times 0.3)}$$

$$= \frac{0.12}{0.42 + 0.12} = \frac{0.12}{0.54} = \frac{2}{9}$$

Conclusion: Therefore, the probability of the second group introduced a new product

$$= \frac{2}{9}.$$

Q. A bag A contains 1 white and 6 red balls. Another bag B contains 4 white and 3 red balls. One of the bags is selected at random, and a ball is drawn from it, which is found to be white. Find the probability that the ball drawn is from bag A.

Ans: Let,

E_1 = event that ball drawn is red ~~from bag A~~

E_2 = event that ball drawn is white ~~from bag B~~

E_3 = event that ~~ball drawn is red from~~ bag B selected

E_4 = event that ~~ball drawn is white from~~ bag A selected

~~$P(E_1) =$~~

$P(E_3) = P(E_4) = \frac{1}{2}$ { assuming, selecting bags is of equal probability i.e. $\frac{1}{2}$

$P(E_2 | E_3)$

~~$P(E_3 | E_2)$~~ = probability that bag B is selected and ~~red~~ white ball is drawn

$$= \frac{4}{7}.$$

$P(E_2 | E_4)$

~~$P(E_4 | E_2)$~~ = probability that bag A is selected and white ball is drawn

$$= \frac{1}{7}.$$

by using baye's theorem, we get

$P(E_4 | E_2)$ = probability that selected white ball is from bag A.

$$= \frac{P(E_4) \cdot P(E_2 | E_4)}{P(E_3) \cdot P(E_2 | E_3) + P(E_4) \cdot P(E_2 | E_4)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{7}}{\frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{1}{7}} = \frac{1}{5}$$

Conclusion: Therefore, the probability of selected white ball is from bag A.

$$= \frac{1}{5}$$

Q. There are two bags I and II. The bag I contains 3 white and 4 black balls, and bag II contains 5 white and 6 black balls. One ball is drawn at random from one of the bags and is found to be white. Find the probability that it was drawn from the bag I.

Ans: मान लिया कि :-

W : white ball is drawn.
B : Black ball is drawn.
X : 1st bag is selected.
Y : 2nd bag is selected.

Assuming, selecting bags is equal probability i.e. $\frac{1}{2}$

$$P(X) = P(Y) = \frac{1}{2}.$$

$P(W|X)$ = probability that white ball is drawn when ~~2nd~~ 1st bag is selected

$$= \frac{3}{7}.$$

(P.T.O.)

$P(W|Y)$ = probability that white ball is drawn when 2nd bag is selected.

$$= \frac{5}{11}.$$

by using baye's theorem, we get

$P(X|W)$: probability of selected white ball is from the 1st bag.

$$P(X|W) = \frac{P(X) \cdot P(W|X)}{P(X) \cdot P(W|X) + P(Y) \cdot P(W|Y)}$$

$$= \frac{\left(\frac{1}{2}\right) \times \left(\frac{3}{7}\right)}{\left(\frac{1}{2}\right)\left(\frac{3}{7}\right) + \left(\frac{1}{2}\right) \times \left(\frac{5}{11}\right)}$$

$$= \frac{\frac{3}{7}}{\frac{3}{7} + \frac{5}{11}}$$

$$= \frac{33}{68}.$$

Conclusion :- Therefore, the probability of selected white ball is from the 1^{st} bag is $\frac{33}{68}$.

Q. A box contains 2 gold and 3 silver coins. Another box contains 3 gold and 3 silver coins. A box is chosen at random, and a coin is drawn from it. If the selected coin is a gold coin, find the probability that it was drawn from the second box.

Ans: मान लिया कि :—

G : Gold coin is ~~selected~~ drawn.
S : Silver coin is ~~selected~~ drawn.
A : 1^{st} box is choosen.
B : 2^{nd} box is choosen.

Assuming, selecting bags is of equal probability i.e. $\frac{1}{2}$.

So, $P(A) = P(B) = \frac{1}{2}$.

$P(G|A)$ = Probability that a gold coin is drawn when 1^{st} box is chosen.

$= \frac{2}{5}$

(P.T.O.)

P(G|B) = probability that a gold coin is drawn when 2nd box is chosen.

$$= \frac{3}{6} = \frac{1}{2}.$$

by using baye's theorem, we get

P(B|G) = Probability that gold coin is selected from the 2nd box.

$$= \frac{P(B) \cdot P(G|B)}{P(A) \cdot P(G|A) + P(B) \cdot P(G|B)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{2}{5} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}}$$

$$= \frac{5}{9}$$

Conclusion: Therefore, the probability of selected gold coin is from the 2nd box

$$= \frac{5}{9}.$$

Q. Three urns A, B and C contains 6 red and 4 white; 2 red and 6 white; and 1 red and 5 white balls respectively. An urn is chosen at random, and a ball is drawn. If the ball drawn is found to be red, find the probability that the balls was drawn from the first urn A.

Ans: माना लिया कि :-

A: event that urn A is choosen.
B: event that urn B is choosen.
R: event that red ball is drawn.
W: event that white ball is drawn.
C: event that urn C (urn) is choosen.

let's assume that, choosing ~~bag~~ is of equal probabilitity i.e. $\frac{1}{2}$.

$$P(A) = P(B) = \frac{1}{3} = P(C)$$

P(R|A) = Probability that the red ball is drawn when urn A is choosen.

$$= \frac{6}{10}$$

P(R|B) = Probability that the red ball is drawn when urn B is choosen.

$$= \frac{2}{8}.$$

(P.T.O.)

P(R|C) = Probability that the red ball is choosen when urn C is choosen.

$$= \frac{1}{6}$$

Now, by using baye's theorem; we get

P(A|R) = Probability that red ball is selected from urn A.

$$= \frac{P(A) \cdot P(R|A)}{P(A) \cdot P(R|A) + P(B) \cdot P(R|B) + P(C) \cdot P(R|C)}$$

$$= \frac{\left(\frac{1}{2} \times \frac{6}{10}\right)}{\left(\frac{1}{2} \times \frac{6}{10}\right) + \left(\frac{1}{2} \times \frac{2}{8}\right) + \left(\frac{1}{2} \times \frac{1}{6}\right)}$$

$$= \frac{\frac{3}{10}}{\frac{3}{10} + \frac{1}{8} + \frac{1}{12}}$$

$$= \frac{\frac{3}{10}}{\frac{3\times 12 +}{120}}$$

$$= \frac{\left(\frac{1}{3}\times\frac{6}{10}\right)}{\left(\frac{1}{3}\right)\times\left(\frac{6}{10}\right)+\left(\frac{1}{3}\right)\times\left(\frac{2}{8}\right)+\left(\frac{1}{3}\right)\times\left(\frac{1}{6}\right)}$$

$$= \frac{\frac{3}{5}}{\frac{3}{5}+\frac{1}{4}+\frac{1}{6}} = \frac{36}{61}$$

Conclusion: Therefore, the probability of selected red ball is from bag A is

$$= \frac{36}{61}.$$

Q. Three urns contains 2 white and 3 black balls; 3 white and 2 black balls, and 4 white and 1 black ball respectively. One ball is drawn from an urn chosen at random, and it was found to be white. Find the probability that it was drawn from the first urn.

(P.T.O.)

~~Ans~~: मान लिया कि :-

A : First urn is selected.
B : Second urn is selected.
C : Third urn is selected.
BB : black ball is drawn.
WB : white ball is drawn.

Assuming, selecting bags is of equal probability i.e. $\frac{1}{3}$

So,

$$P(A) = P(B) = P(C) = \frac{1}{3}.$$

~~Now, using bayes' theorem, we get~~

P(WB|A) = Probability that white ball is drawn when first urn is selected.

$$= \frac{2}{5}.$$

P(WB|B) = Probability that white ball is drawn when second urn is selected.

$$= \frac{3}{5}.$$

$P(WB|C)$ = Probability that white ball is drawn when third urn is selected.

$$= \frac{4}{5}.$$

Now, by using baye's theorem; we get

$$P(A|WB) = \frac{P(A)\cdot P(WB|A)}{P(A)\cdot P(WB|A) + P(B)\cdot P(WB|B) + P(C)\cdot P(WB|C)}$$

$$= \frac{\left(\frac{1}{3}\right)\times\left(\frac{2}{5}\right)}{\left(\frac{1}{3}\right)\cdot\left(\frac{2}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{5}\right) + \left(\frac{1}{3}\right)\cdot\left(\frac{4}{5}\right)}$$

$$= \frac{2}{9}.$$

Conclusion: Therefore, the probability of selected white ball is from bag A.

$$= \frac{2}{9}.$$

Q. There are three boxes, the first one containing 1 white, 2 red and 3 black balls; the second one containing 2 white, 3 red and 1 black ball and the third one containing 3 white, 1 red & 2 black balls. A box is chosen at random, and from it, two balls are drawn at random. One ball is red and the other, white. What is the probability that they

come from the second box ?

Ans: मान लिया कि :—

A : bag A is selected.
B : bag B is selected.
C : bag C is selected.
BB : black ball is drawn.
W[redacted] : white ball is drawn.
R[redacted] : red ball is drawn.

Assuming, selecting the bag is of equal probability i.e. $\frac{1}{3}$

So,
$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$P(WR|A)$ = probability that white and red ball is drawn when bag A is selected.

$$= \frac{3}{56}$$

$P(WR|B)$ = probability that white and red ball is drawn when bag B is selected

$$= \frac{5}{6}$$

$P(WR|C)$ = probability that white and red ball is drawn when bag C is selected

$$= \frac{4}{6} = \frac{2}{3}$$

Now, by using Baye's theorem, we get

$$P(B|WR) = \frac{P(B) \times P(WR|B)}{P(A) \cdot P(WR|A) + P(B) \cdot P(WR|B) + P(C) \cdot P(WR|C)}$$

$$= \frac{\frac{1}{3} \times \frac{5}{6}}{\frac{1}{3} \times \frac{3}{6} + \frac{1}{3} \times \frac{5}{6} + \frac{1}{3} \times \frac{2}{3}}$$

$$= \frac{\frac{5}{18}}{\frac{3}{18} + \frac{5}{18} + \frac{2}{9}}$$

$$= \frac{\frac{5}{18}}{\frac{6+10+8}{36}}$$

$$= \frac{5 \times 36}{24 \times 18} = \frac{5}{12}$$

Conclusion: Therefore,

the probability of selected white and red ball from bag B

$$= \frac{5}{12}.$$

Q. Urn A contains 7 white and 3 black balls; urn B contains 4 white and 6 black balls; urn C contains 2 white and 8 black balls. One of these urns is chosen at random with probabilities 0.2, 0.6 and 0.2 respectively. From the chosen urn, two balls are drawn at random without replacement. Both the balls happen to be white. Find the probability that the balls are drawn from urn C.

Ans: मान लिया कि :-

A: bag A is choosen
B: bag B is choosen
C: bag C is choosen
Z: black ball is drawn
X: white ball is drawn
Y: red ball is drawn

(P.T.O.)

given,

$P(A) = 0.2$; $P(B) = 0.6$; $P(C) = 0.2$

$P(X|A)$ = Probability that white ball is drawn when bag A is choosen.

$$= \frac{{}^{7}C_2}{{}^{10}C_2}$$

(No. of ways of choosing/drawing 2 balls out of 7 white balls)

(No. of ways of drawing 2 ball out of total 10 balls in urn A)

$P(X|B)$ = Probability that white ball is drawn when bag B is choosen.

$$= \frac{{}^{4}C_2}{{}^{10}C_2}$$

(No. of ways of drawing 2 balls out of 4 white balls)

(No. of ways of drawing 2 balls out of 10 balls in total from urn B)

$P(X|C)$ = Probability that white ball is drawn when bag C is choosen.

$$= \frac{{}^{2}C_2}{{}^{10}C_2}$$

(No. of ways of drawing 2 balls out of 2 white balls available ~~in total~~ in bag C)

(No. of ways of drawing 2 balls out of 2 ~~white~~ balls from 10 available ball in urn C)

Now, using baye's theorem, we get

P(C | X) = Probability that white ball is drawn when bag C is choosen.

$$= \frac{P(C) \cdot P(X|C)}{P(A) \cdot P(X|A) + P(B) \cdot P(X|B) + P(C) \cdot P(X|C)}$$

$$= \frac{0.2 \times \frac{1}{45}}{0.2 \times \frac{21}{45} + 0.6 \times \frac{6}{45} + 0.2 \times \frac{1}{45}}$$

$$= \frac{1}{40}$$

Conclusion:

Therefore, the probability of both selected white balls are from urn C is $\frac{1}{40}$.

(P.T.O.)

Q. There are 3 bags, each containing 5 white and 3 black balls. Also, there are 2 bags, each containing 2 white and 4 black balls. A white ball is drawn at random. Find the probability that this ball is from a bag of the first group.

Ans: मान लिया कि :-

A: set of first 3 bags.
B: a set of next 2 bags.
W: white ball is drawn.
Z: black ball is drawn.

अब हम problem को change करके दो bag का बना देते हैं i.e. bag A में 15 white और 9 black balls हैं (5 white & 3 black in each bag) and bag B में 4 white and 8 black balls हैं (2 white & 4 black in each bag)

अब,

Probability of selecting bag A $= \frac{3}{5}$ (3 bags are in A)

Probability of selecting bag B $= \frac{2}{5}$ (2 bags are in B)

अब हमको निकालना है

P(W|A) = Probability that white ball is drawn when the first set of 3 bags are taken.

$$= \frac{15}{24}$$

$P(W|B)$ = Probability that the white ball is drawn when the next set of 2 bags are taken

$$= \frac{4}{12}$$

Now, using the baye's theorem, we get

$P(A|W)$ = Probability that the ball is drawn from first set of 3 bag if the ball is white.

$$= \frac{P(A) \cdot P(A|W)}{P(A) \cdot P(A|W) + P(B) \cdot P(B|W)}$$

$$= \frac{\left(\frac{3}{5}\right) \times \left(\frac{15}{24}\right)}{\left(\frac{3}{5}\right) \times \left(\frac{15}{24}\right) + \left(\frac{2}{5}\right) \times \left(\frac{4}{12}\right)}$$

$$= \frac{45}{61}.$$

Conclusion:

Therefore, the probability of selected white ball is from the first group is

$$= \frac{45}{61}$$

Q. There are four boxes A, B, C and D, containing marbles. A contains 1 red, 6 white and 3 black marbles; B contains 6 red, 2 white and 2 black marbles; C contains 8 red, 1 white and 1 black marbles; and D contains 6 white and 4 black marbles. One of the boxes is selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from the box A?

Ans: मान लिया कि :—

A: event that bag A is choosen.
B: event that bag B is choosen.
C: event that bag C is choosen.
D: event that bag D is choosen.
X: event that ball drawn is black.
Y: event that ball drawn is white
Z: event that ball drawn is red.

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Assuming all boxes have equal probability for picking i.e. $\frac{1}{4}$.

So,

$P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$.

$P(Z|A) =$ Probability that ball drawn is red if bag A is choosen

$= \frac{1}{10}$

$P(Z|B) =$ Probability that ball drawn is red if bag B is choosen

$= \frac{6}{10}$

$P(Z|C) =$ Probability that ball drawn is red if bag C is choosen

$= \frac{8}{10}$

$P(Z|D) =$ Probability that ball drawn is red if bag D is choosen

$= \frac{0}{10}$.

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by, using baye's theorem :-

P(A|Z) = Probability that ball drawn is red from the box A

$$= \frac{P(A)\cdot P(Z|A)}{P(A)\cdot P(Z|A) + P(B)\cdot P(Z|B) + P(C)\cdot P(Z|C) + P(D)\cdot P(Z|D)}$$

$$= \frac{\left(\frac{1}{4}\right)\times\left(\frac{1}{10}\right)}{\left(\frac{1}{4}\right)\cdot\left(\frac{1}{10}\right) + \left(\frac{1}{4}\right)\times\left(\frac{6}{10}\right) + \left(\frac{1}{4}\right)\left(\frac{8}{10}\right) + \left(\frac{1}{4}\right)\times\left(\frac{0}{10}\right)}$$

$$= \frac{1}{15}$$

Conclusion: Therefore, the probability of selected red ball is from box A is $\frac{1}{15}$

Q. A car manufacturing factory has two plants X and Y. Plant X manufactures 70% of the cars, and plant Y manufactures 30%. At plant X, 80% of the cars are rated of standard quality. A car is picked up at random and is found to be of standard quality. Find the probability that it has come from plant X.

& at plant Y 90% rated standard quality

Ans: मान लिजिए :-

X: Car produced from plant X.
Y: Car produced from plant Y.
S: Car rated as standard quality.

$$P(X) = \frac{70\%}{100\%} = \frac{70/100}{100/100} = \frac{70}{100}$$

$$P(Y) = \frac{30\%}{100\%} = \frac{30/100}{100/100} = \frac{30}{100}$$

$P(S|X)$ = Probability that the car rated as standard quality is produced from plant X.

$$= \frac{80\%}{100\%} = \frac{80/100}{100/100} = \frac{80}{100}$$

P(S|Y) = Probability that car rated as standard quality produced in plant Y.

$$= \frac{90\%}{100\%} = \frac{90/100}{100/100} = \frac{90}{100}$$

Now, by using bayes's theorem, we get

P(X|S) = Probability that a car produced from plant X is of standard quality

$$= \frac{P(X) \cdot P(S|X)}{P(X) \cdot P(S|X) + P(Y) \cdot P(S|Y)}$$

$$= \frac{\left(\frac{70}{100}\right)\left(\frac{80}{100}\right)}{\left(\frac{70}{100}\right) \times \left(\frac{80}{100}\right) + \left(\frac{30}{100}\right) \times \left(\frac{90}{100}\right)}$$

$$= \frac{56}{83}.$$

Conclusion: Therefore, the probability of selected standard quality car is from plant X is

$$\frac{56}{83}.$$

Q. An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0.01, and that of motorcycles is 0.02. An insured vehicle met with an accident. Find the probability that the accidented vehicle was a motorcycle.

Ans: मान लिया कि :-

M : Motorcycle
S : Scooter
A : Accidented vehicle.

$$P(M) = \frac{3000}{5000} \quad ; \quad P(S) = \frac{2000}{5000}$$

$P(A|M)$ = probability that accidented vehicle was a motorcycle

$= 0.02$

$P(A|S)$ = probability that accidented vehicle was a scooter

$= 0.01.$

Conclusion: Therefore, the probability of selected standard quality car is from plant X.

$= \frac{56}{83}$

by, baye's theorem, we get :-

P(

$P(S|Y)$ = Probability that the car rated as standard quality is produced from plant Y.

$$= \frac{90\%}{100\%} = \frac{90/100}{100/100} = \frac{90}{100}.$$

by using baye's theorem, we get

$P(X|S)$ = Probability that car produced from plant X is of standard quality.

$$= \frac{P(X) \cdot P(S|X)}{P(X) \cdot P(S|X) + P(Y) \cdot P(S|Y)}$$

$$= \frac{\left(\frac{70}{100}\right) \times \left(\frac{80}{100}\right)}{\left(\frac{70}{100}\right)\left(\frac{80}{100}\right) + \left(\frac{30}{100}\right) \times \left(\frac{90}{100}\right)} = \frac{56}{83}$$

Conclusion: Therefore, the probability that selected standard quality car is from plant X

$$= \frac{56}{83}.$$

Q. An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0.01, and that of motorcycles is 0.02. An insured vehicle met with an accident. Find the probability that the accidented vehicle was a motorcycle.

Ans: दिया गया है :—

M : Motorcycle.
S : scooter
A : Accidented vehicle.

$$P(M) = \frac{3000}{5000} \qquad P(S) = \frac{2000}{5000}$$

$P(A|M)$ = Probability that accidented vehicle was a motorcycle.

$$= 0.02$$

(P.T.O.)

P(A|S) = Probability that accidented vehicle was a scooter.

= 0.01.

by using bayes theorem, we get

P(M|A) = Probability that accidented vehicle was a motorcycle.

$$= \frac{P(M) \cdot P(A|M)}{P(M) \cdot P(A|M) + P(S) \cdot P(A|S)}$$

$$= \frac{\left(\frac{3000}{5000}\right) \times \frac{(0.02)}{100}}{\left(\frac{3000}{5000}\right) \times \frac{0.02}{100} + \left(\frac{2000}{5000}\right) \times \frac{0.01}{100}}$$

$$= \frac{6}{8} = \frac{3}{4}$$

Conclusion: Therefore,

the probability of accidented vehicle was a motorcycle

$$= \frac{3}{4}.$$

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Q. In a bulb factory, machines A, B and C manufactures 60%, 30% and 10% bulbs respectively. Out of these bulbs 1%, 2% and 3% of the bulbs produced respectively by A, B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by machine A.

Ans: माना लिया कि :–

A: Manufactured from machine A.
B: Manufactured from machine B.
C: Manufactured from machine C.
D: Defective bulb.

$$P(A) = \frac{60\%}{100\%} = \frac{60/\cancel{100}}{100/\cancel{100}} = \frac{60}{100}$$

$$P(B) = \frac{30\%}{100\%} = \frac{30/\cancel{100}}{100/\cancel{100}} = \frac{30}{100}$$

$$P(C) = \frac{10\%}{100\%} = \frac{10/\cancel{100}}{100/\cancel{100}} = \frac{10}{100}$$

P(D|A) = Probability that a defective bulb is manufactured from Machine A.

$$= 1\% = \frac{1}{100}$$

$P(D|B)$ = Probability that a defective bulb is manufactured from Machine B.

$$= 2\% = \frac{2}{100}$$

$P(D|C)$ = Probability that a defective bulb is manufactured from Machine C.

$$= 3\% = \frac{3}{100}.$$

therefore; by baye's theorem, we get

$P(A|D)$ = Probability that a defective bulb is manufactured from machine A.

$$= \frac{P(A) \cdot P(D|A)}{P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D|C)}$$

$$= \frac{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right)}{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right) + \left(\frac{30}{100}\right)\left(\frac{2}{100}\right) + \left(\frac{10}{100}\right)\left(\frac{3}{100}\right)}$$

$$= \frac{6}{15} = \frac{2}{5}$$